# Is there an Optimal Level of Globalization?

## Stochastic Fragility Dynamics with Trend Reinforcing

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September 22, 2007

# Outline

# Bankruptcy Risk

- What to do against bankruptcy risk?
  - Hedge risk
    - Diversify risk
    - Share risk
  - Essentially: connect with other firms → **globalization**
- But globalization also seems to destabilize economy
  - Correlations of bad evolution
  - Hedge funds are suspect to destabilize
- Is there an optimal hedging level?
  - In a **connected** economy
  - In a **trend reinforcing** economy

# Fragility of a firm

- Fragility should give a measure of distance to bankruptcy
- It raises e.g. when
  - liabilities go beyond equity (measured as net worth)
  - sales volume decreases faster then fixed costs (measured as profit)
- Applicable for producing firms, banks, hedge funds, ...
- If **fragility hits a threshold** then the **firm goes bankrupt**

# Bankruptcy risk, avalanches and systemic risk

- **Systemic risk** is the risk that a huge part of the ecomony goes bankrupt for endogeneous reasons
  - Difficult to quantify
- If a failing firm causes additional fragility to connected firms, this could cause an **avalanche** of bankrupcies
- A higher **bankruptcy risk** could have huge impact on systemic risk
  - could serve as a first proxy for systemic risk

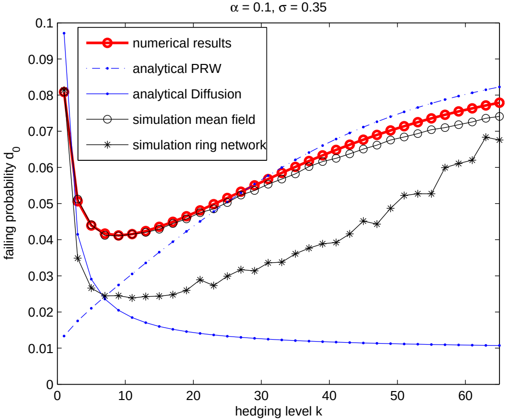
# Trend information and fragility

- Information about a firm (from stock prices, yearly reports or S&P credit rank) is often reduced to either
  - Firm is performing good (better than average, $\nearrow$, BUY), or
  - Firm is performing bad (worse than average, $\searrow$, SELL)
- Can induce positive feedback by additional costs/benefits
  - $\nearrow \rightsquigarrow$ better excess to capital, better interest rates $\rightsquigarrow \nearrow\nearrow$
  - $\searrow \rightsquigarrow$ removal of capital, bad credit conditions $\rightsquigarrow \searrow\searrow$

# Risk hedging and fragility

- We assume that firms try to hedge their bankruptcy risk with other firms, e.g. by
  - Insurances
  - Long term supply contracts
  - Doing like other firms do (if they perform well)
- This implies that a firm's fragility evolves as an **average** of the fragilities of all its hedging partners
- **How does the hedging network effect the bankruptcy risk?**

# Is there an optimal level of connectivity?




- The final figure in advance → Yes, in the following fragility model

# The model

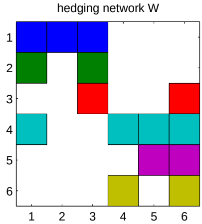
- $n$ firms
- $\phi_i(t) \in [0, \theta]$ fragility of firm $i$ at time $t \in \mathbb{N}$
- $\theta$ failing threshold (normalized to 1 w.l.o.g.)
- $\phi(0) \in [0, \theta]^n$ vector of initial fragilities
- Further ingredients
  - stochastic process
  - hedging network
  - trend reinforcing

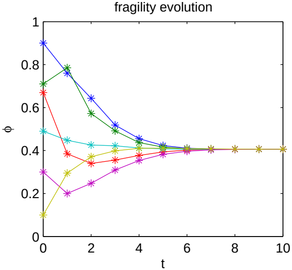
# Hedging network averages fragilities

- Firms are in a hedging network $W \in \mathbb{R}^{n \times n}$
- $w_{ij} \leq 0$ and $\sum_{j=1}^{\infty} w_{ij} = 1$
- Hedging is performed as

$$\phi(t+1) = W\phi(t)$$

- $\phi_i(t+1)$ is a weighted mean of other fragilities



# Stochastic shocks and trend reinforcing

- Fragility change by additive stochastic shocks $\xi$ with $E(\xi) = 0$, $\mathrm{std}(\xi) = \sigma$, no skewness (e.g. Gaussian)
- Trend reinforcing by adding trend strength $\alpha$ with the sign of the trend $\Delta\phi_i = \phi_i(t) - \phi_i(t-1)$
- Dynamical equation

$$\phi(t+1) = W(\phi(t) + \sigma\xi(t)) + \alpha \mathrm{sign}(W(\phi(t) - \phi(t-1)))$$

# Boundaries: Lowest fragility and bankruptcies

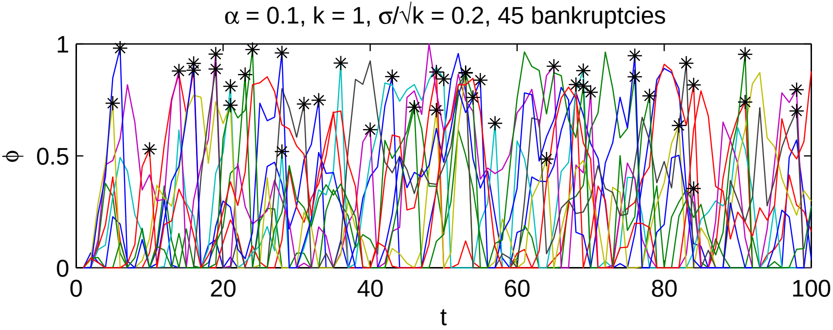
- Lower bound zero
- Upper bound $\theta$: Firms above go bankrupt and get replaced by new firm with zero fragility.
- Dynamical equation[1]

$$\phi(t+1) = \mathbf{1}_{[0,\theta]}\left(W(\phi(t) + \sigma\xi(t)) + \alpha \mathrm{sign}(W(\phi(t) - \phi(t-1)))\right)$$

[1] Battiston, S., Delli Gatti, D., Gallegati, M., 2007

# Example

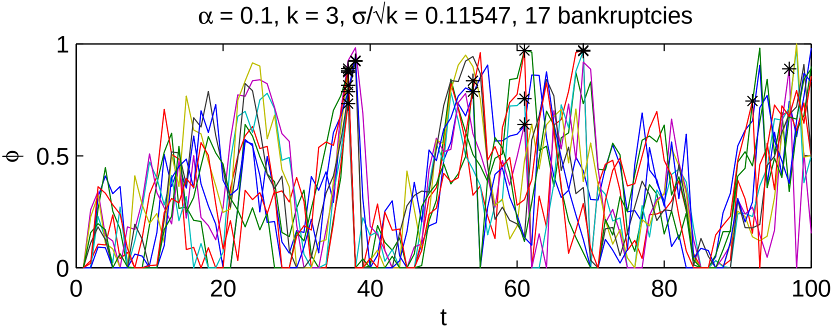
$$\phi(t+1) = \mathbf{1}_{[0,\theta]}\left(W(\phi(t) + \sigma\xi(t)) + \alpha \mathrm{sign}(W(\phi(t) - \phi(t-1)))\right)$$




- No hedging (0 neighbor)

# Example

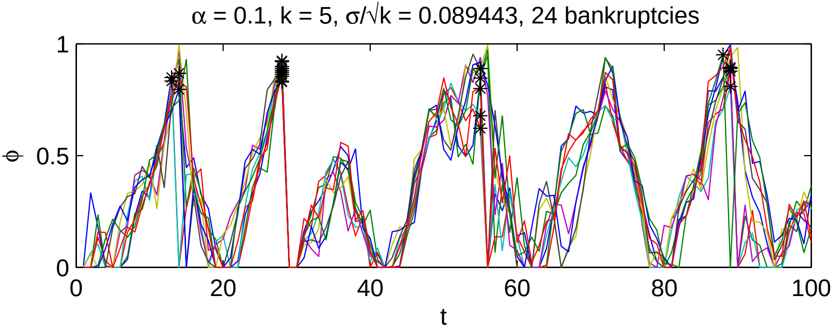
$$\phi(t+1) = \mathbf{1}_{[0,\theta]}\left(W(\phi(t)+\sigma\xi(t)) + \alpha\mathrm{sign}(W(\phi(t)-\phi(t-1)))\right)$$




- Some connections (2 neighbors) decrease bankruptcy risk

# Example

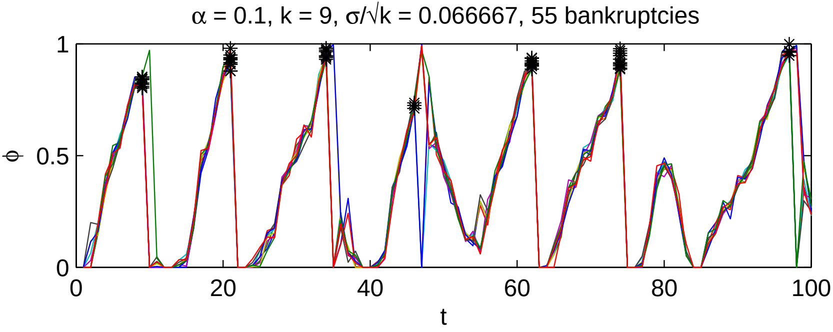
$$\phi(t+1) = \mathbf{1}_{[0,\theta]}\left(W(\phi(t) + \sigma\xi(t)) + \alpha \mathrm{sign}(W(\phi(t) - \phi(t-1)))\right)$$




- More connections (4 neighbors) correlate evolutions

# Example

$$\phi(t+1) = \mathbf{1}_{[0,\theta]}\left(W(\phi(t) + \sigma\xi(t)) + \alpha \mathrm{sign}(W(\phi(t) - \phi(t-1)))\right)$$




- Even more connections (6 neighbors) increase bankruptcy risk!

# Higher number of links decreases $\sigma$ in the mean field limit

- $\bar{\phi}(t)$ average fragility at time $t$
- $k$ average number of hedging partners of each firm
- Mean field dynamics

$$\bar{\phi}(t+1) = \bar{\phi}(t) + \frac{\sigma}{\sqrt{k}}\xi(t) + \alpha \text{sign}(\Delta\bar{\phi}(t)).$$

- Increasing the number of hedging partners $k$ decreases the std of shocks $\sigma$ by a factor $\sqrt{k}$.
  In the following the specific network is neglected!

# Probability to keep the trend

- Define $\Delta\phi(t) = \phi(t) - \phi(t-1)$ as the **trend** $\mathrm{tr}(t) \in \{-1, +1\}$
- Trend keeping probability is

$$q(\alpha,\sigma) = \Pr(\sigma\xi < \alpha) = \int_{-\infty}^{\frac{\alpha}{\sigma}} f_\xi(x)dx$$

- So, $\mathrm{tr}(t) \in \{-1, +1\}$ and

$$\Pr(\mathrm{tr}(t+1) = +1 \mid \mathrm{tr}(t) = +1) = q$$
$$\Pr(\mathrm{tr}(t+1) = -1 \mid \mathrm{tr}(t) = -1) = q$$
$$\Pr(\mathrm{tr}(t+1) = -1 \mid \mathrm{tr}(t) = +1) = 1-q$$
$$\Pr(\mathrm{tr}(t+1) = +1 \mid \mathrm{tr}(t) = -1) = 1-q$$

# Reformulation of the process

$$\bar{\phi}(t+1) = \mathbf{1}_{[0,\theta]}(\bar{\phi}(t) + \sigma\xi(t) + \alpha \mathrm{tr}(t))$$

- with

$$\mathrm{tr}(t+1) = \begin{cases} 0 & \text{if } \bar{\phi}(t+1) = 0 \\ \text{otherwise} & \begin{cases} -1 & \text{if } \mathrm{tr}(t) = 0 \text{ and } \sigma\xi(t) < 0 \\ 1 & \text{if } \mathrm{tr}(t) = 0 \text{ and } \sigma\xi(t) > 0 \\ -\mathrm{tr}(t) & \text{if } \mathrm{tr}(t) = 1 \text{ and } \sigma\xi(t) < -\alpha \\ -\mathrm{tr}(t) & \text{if } \mathrm{tr}(t) = -1 \text{ and } \sigma\xi(t) > \alpha \\ \mathrm{tr}(t) & \text{otherwise} \end{cases} \end{cases}$$

- Trend **persists** with probability $q$

# Trend produces persistent random walk (PRW)

$$\bar{\phi}(t+1) = \bar{\phi}(t) + \mathrm{tr}(t)$$

- First ballistic scaling $\mathrm{std}(\phi(t)) \propto t$
- For $t \to \infty$ the central limit theorem prevails with $\mathrm{std}(\phi(t)) \to (t\frac{q}{1-q})^{\frac{1}{2}}$
- Obeys telegrapher's equation

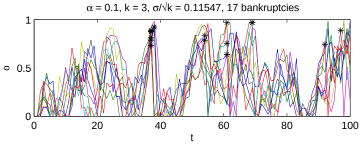
# Ingredients of the process

$$\bar{\phi}(t+1) = \mathbf{1}_{[0,\theta]}(\bar{\phi}(t) + \underbrace{\sigma\xi(t)}_{\text{diffusive scaling}} + \underbrace{\alpha \mathrm{tr}(t)}_{\text{ballistic} \to \text{diffusive}})$$

- Parameters of interest
  - noise level $\sigma$
  - trendstrength $\alpha$
  - (failing threshold $\theta$)
- Derived variable
  - trend keeping probability $q = F_\xi(\frac{\alpha}{\sigma})$

# Mean first passage time and bankruptcy probability

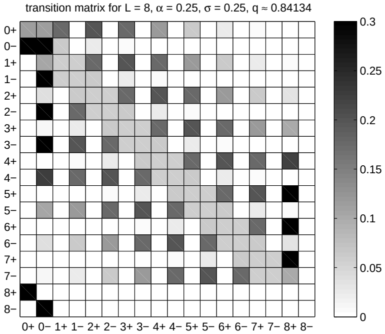
- Define
  - $m_0$ the mean time to bankruptcy starting in zero with random trend
  - $\langle m \rangle$ mean first passage time starting anywhere with equal probability and random trend




- Analog
  - $d_0 = \frac{1}{m_0}$ average bancruptcy probability if processes restart at zero
  - $d_{\langle m \rangle} = \frac{1}{\langle m \rangle}$ average bancruptcy probability if processes restart anywhere

# Approximation as discrete Markov chain

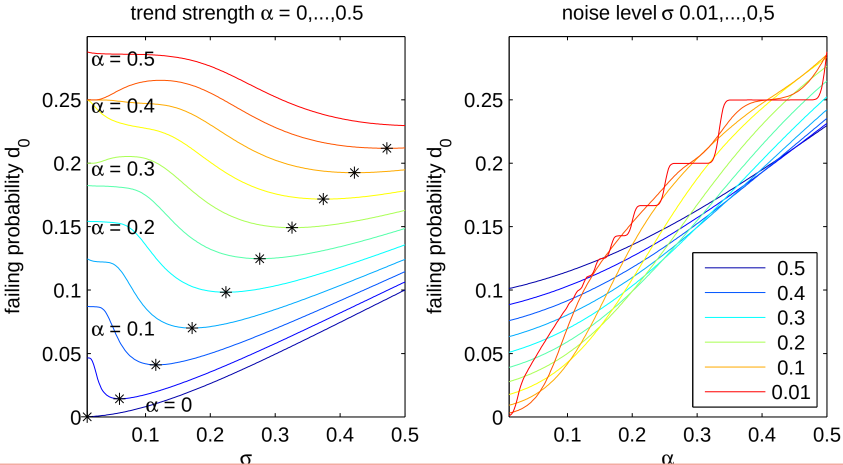
- The transition matrix for discretization to $L$ states is $P =$




- Solve the linear system $(I - Q)m = 1$, then $m$ is vector of mean first passage times ($Q$ is $P$ without the last two classes)

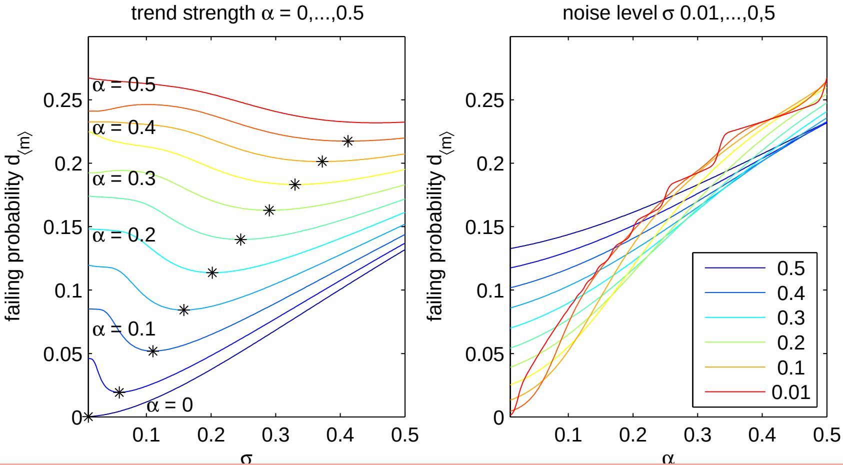
# Bankruptcy probability $d_0$

- Fixed trend strength $\alpha \to$ **optimal noise level** $\sigma$ exists

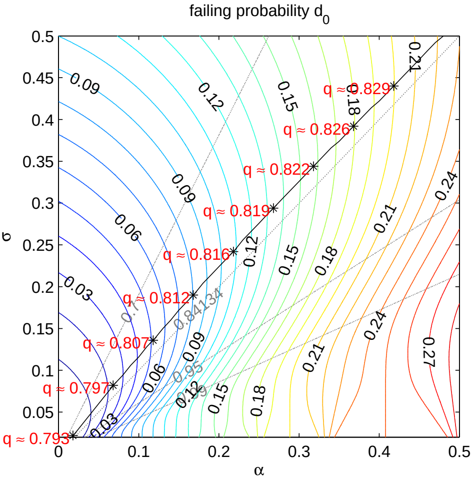


# Bankruptcy probability $d_{\langle m \rangle}$

- Fixed trend strength $\alpha \rightarrow$ **optimal noise level** $\sigma$ exists

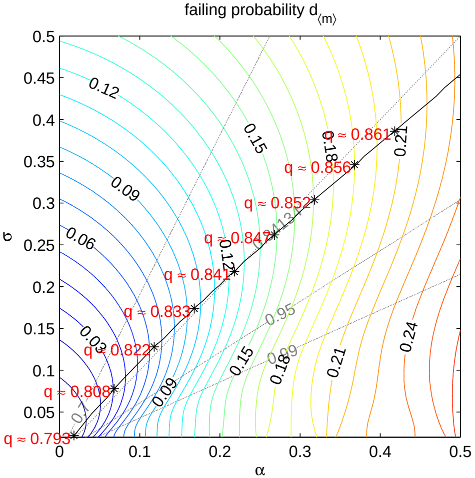


# Bankruptcy probability $d_0$ and trend keeping probability $q$



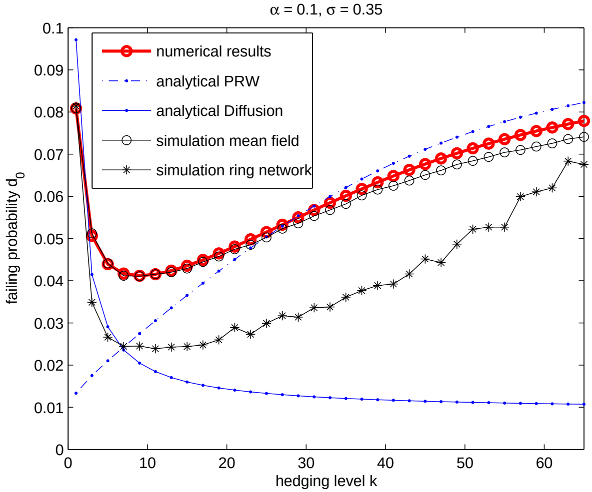

The minimal bankruptcy probability for fixed $\alpha$ appears at a certain value of $q(\alpha, \sigma)$ which decreases with $\frac{\alpha}{\sigma}$

# Bankruptcy probability $d_{\langle m \rangle}$ and trend keeping probability $q$




The minimal bankruptcy probability for fixed $\alpha$ appears at a certain value of $q(\alpha, \sigma)$ which decreases with $\alpha, \sigma$

# There is an optimal level of globalization




- Analytical expression for pure presistent random walk

$$m_0 = \frac{L(L-1)}{q} - ((L-1)(L-3)-2) - 2(L-1)q$$

# Variations and robustness of results

- Variations
  - Other methods of firm's rebirth
  - Other reflection modes at zero
- These Variations change results quantitatively but not qualitatively

- We expect great impact of the network structure beyond the mean field assumption
- Other types of trend reinforcing are possible, robustness of results is not clear

# Summary

- Trend strength $\alpha$
  - Represents degree how the economic climate fosters positve feedback
  - Higher trend strength always leads to higher bankruptcy risk
- Improve risk hedging $\frac{\sigma}{\sqrt{k}}$
  - Increasing the average number of hedging partners $k$ decreases the size of the shocks $\sigma\xi$
  - For every trend strength there is a **local minimum** of the bankrupcy risk at a certain $\frac{\sigma}{\sqrt{k}}$
  - There is a certain optimal trend persistence probability $q$
- This model suggests that an **intermediate average** connectivity/**globalization** is optimal to **reduce the bankruptcy probability** and succesively the systemic risk of a large break-down

# References

Basic equation:

- Battiston, S., Delli Gatti, D., Gallegati, M., 2007, *Emergence of Systemic Risk in Trade Credit Network*, to appear in Helbing, D. ed., Managing Complexity, Springer

In preparation:

- Lorenz J., Battiston S., 2007, *Mean first passage time and bankruptcy probability in a trend-reinforcing economy*
- Lorenz J., Battiston S., 2007, *The optimal level of globalization under stochastic fragility dynamics with trend persistence*