

# Bounded Confidence Simulation Models in Politics: How parties might evolve, drift and unite

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Discussion"

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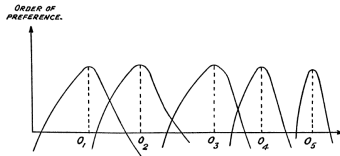
## The spatial model

### Assumptions

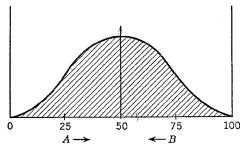
- ▶ Possible policies are on a 1d scale
- ▶ Voters have single-peaked preferences
- ▶ Parties are vote-seeking

### Median Voter Theorem

- ▶ When 2 parties compete, they will both propose policies close to the median voter.
- ▶ The median voter is unbeatable in pairwise polls.

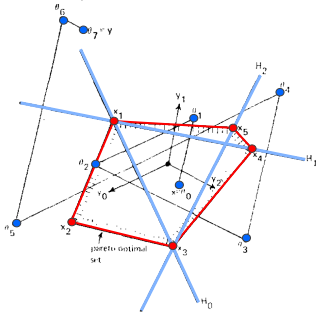


Single-peaked utility functions  
Black, D., On the Rational of Group Decision-making, Journal of Political Economy, 1948



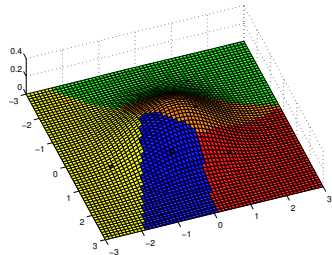
Distribution of voters ideal points  
Downs, A., An Economic Theory of Democracy, 1957

## Two Policy Dimensions, Chaos Theorem, Many Parties



**Chaos Theorem** In 2d policy spaces, the agenda-setter can (almost sure) find a sequence of polls to every policy. No equilibrium.

McKelvey, R., Intransitivities in multidimensional voting models and some implications for agenda control, *Journal of Economic Theory*, 1976

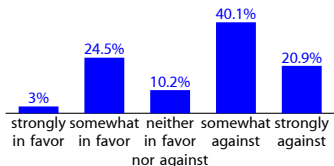


**Adaptive parties in multi-party system**  
Agent-based model with HUNTER, AGGREGATOR, PREDATOR or STICKER strategies for parties. Complex dynamics with few agents.

Laver, M., Policy and the Dynamics of Political Competition, *American Political Science Review*, 2005

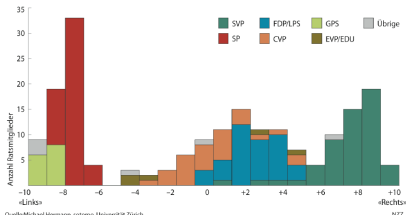
## Empirical Stylized Fact: Bi-, Tri-modal distributions

How much are you in favor or against bailout payments for over-indebted EU countries?



### Survey in Germany among 4,499 persons.

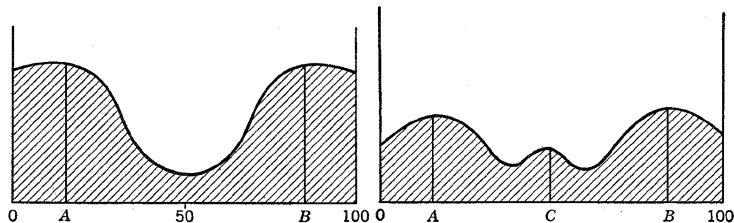
Bechtel, M.M., Hainmueller, J., Margalit, Y.M., Sharing the Pain: What Explains Public Opinion Towards International Financial Bailouts?, MIT Political Science Working Paper, 2012



### Political positions Nationalräte Schweiz

Michael Herrmann, sotomo, Universität Zürich, in NZZ

## Theoretical Necessity: Multimodal distribution of voters' ideal points



Downs, A., *An Economic Theory of Democracy*, 1957

Rational Extremists might abstain (1) as a threat to the closer party to not move central, or because (2) small differences of central parties are indistinguishable.

- ⇒ Many-party systems arise under proportional voting when ideal points are multimodal
- ⇒ Empirical multimodality is likely as many-party systems exist empirically

### How do multimodal landscapes of ideal points form?

## Idea: Multi-agent dynamics of adaptive voters adjusting their ideal points

Adaptive strategies/heuristics for voters

- ▶ Aim to **agree with others** but keep some trust in current position (Coordination Game)

→ utility from minimizing distances to positions of others and own

→ with (singlepeaked) utility functions  $u(y) = -\sum_k (y - x_k)^2$   
myopic agents choose **arithmetic average** as new position

$$y = \frac{1}{\#\{k \mid k \text{ relevant}\}} \sum_k x_k \rightsquigarrow x_i(t+1) = \frac{1}{\#\{k \mid k \text{ relevant}\}} \sum_k x_k(t)$$

Groeber, P., Lorenz, J., Schweitzer, F., A microfoundation of social influence in models of opinion formation, "minor revision" at Journal of Mathematical Sociology, 2012

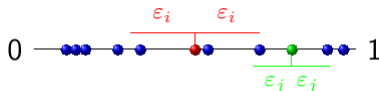
- ▶ Aim to avoid others which are far away (**Homophily**)

Feld, S., Grofman, B., Homophily and the focused organization of ties, Oxford Handbook Analytical Sociology, 2009

→ Agent i chooses agent k as relevant when  $|x_i - x_k| < \varepsilon_i$   
(**bounded confidence**)

## Bounded Confidence Opinion Dynamics Models

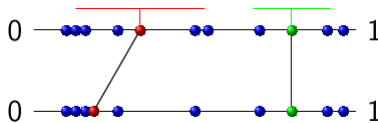
- ▶ N agents (e.g.  $N = 250$ )
- ▶ Each agent starts with an **initial opinion**  $x_i$  between 0 and 1
- ▶ Each agent has an initial **bound of confidence**  $\varepsilon_i > 0$  (e.g.  $\varepsilon_i = 0.2$ )



- ▶ **Bounded Confidence Models:** Each agent is only willing to adjust towards opinions within its area of confidence

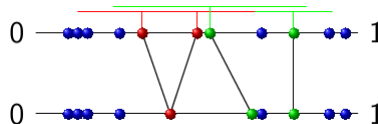
## Two Models: Different Communication Regimes

- Interaction with whole group **Hegselmann-Krause Model**



Hegselmann, R., Krause, U., Opinion Dynamics and Bounded Confidence, Models, Analysis and Simulation, Journal of Artificial Societies and Social Simulation, 2002

- Pairwise interaction **Deffuant-Weisbuch Model**



Deffuant, G., Neau, D., Amblard, F., Weisbuch, G., Mixing Beliefs among Interacting Agents Advances in Complex Systems, 2000



# Simulation

## Netlogo-Model

Lorenz J., Continuous Opinion Dynamics under Bounded Confidence,  
<http://cc1.northwestern.edu/netlogo/models/community/bc>, March 2012

### Dynamics

1. Extermists move towards center
2. Density at extremes rises
3. Dense regions attract agents from center
4. Clusters emerge close to extremes
5. And so on, until the center is reached

## Basic Facts and Questions

- ▶ A stable configuration of opinions emerges
- ▶ The stable configuration is clustered
- ▶ Prerequisite for a simulation (external parameters):
  - ▶ Initial profile of opinions  $x_i(0)$
  - ▶ Bounds of confidence  $\varepsilon_i$
- ▶ What is the impact on the final configuration?
  - ▶ Number of parties
  - ▶ Location of parties
  - ▶ Size of parties

## From agents to densities of agents

Regard density function on the opinion space

$$P(t, \cdot) : [0, 1] \rightarrow \mathbb{R}_{\geq 0}$$

for all  $t \in \mathbb{N}$

$$\int_0^1 P(t, x) dx = 1.$$

## Density evolution in the Hegselmann-Krause model<sup>1</sup>

$$P(t+1, x) = P(t, x) + \int_S \underbrace{\delta(x - M_1(y, P(t, \cdot), \varepsilon)) P(t, y)}_{\text{fraction joining state } x} - \underbrace{\delta(y - M_1(x, P(t, \cdot), \varepsilon)) P(t, x)}_{\text{fraction leaving state } x} dy$$

with  $\varepsilon$ -local barycenter

$$M_1(x, P(t, \cdot), \varepsilon) = \frac{\int_{x-\varepsilon}^{x+\varepsilon} y P(t, y) dy}{\int_{x-\varepsilon}^{x+\varepsilon} P(t, y) dy}.$$

<sup>1</sup> Fortunato et al, IJMPC 2004, Lorenz, MSO Conference, 2004

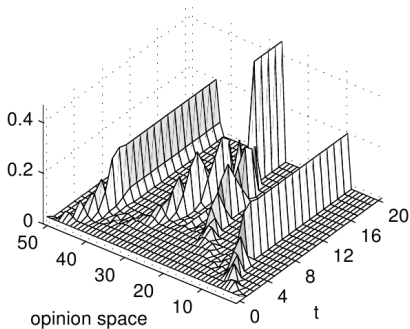
## Density evolution in the Deffuant-Weisbuch model<sup>2</sup>

$$\begin{aligned}
 P(t+1, \mathbf{x}) = P(t, \mathbf{x}) + \int_S \int_{\|\mathbf{x}_1 - \mathbf{x}_2\| \leq \epsilon} P(t, \mathbf{x}_1) P(t, \mathbf{x}_2) & \underbrace{\left( \delta\left(\mathbf{x} - \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}\right) \right)}_{\text{fraction joining state } \mathbf{x}} \\
 - \underbrace{\left( \delta(\mathbf{x} - \mathbf{x}_1) + \delta(\mathbf{x} - \mathbf{x}_2) \right)}_{\text{fraction leaving state } \mathbf{x}} & d\mathbf{x}_2 d\mathbf{x}_1
 \end{aligned}$$

<sup>2</sup> Ben-Naim et al, Physica D 2003

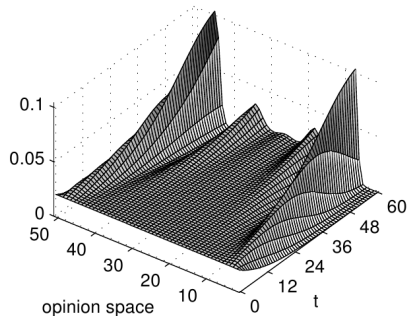
## Computation with discrete approximation<sup>3</sup>

HK model,  $n = 51$ ,  $\varepsilon = 5$ ,  $\varepsilon/n \oplus 0.1$



► HK model converges in finite time

DW model,  $n = 51$ ,  $\varepsilon = 5$ ,  $\varepsilon/n \oplus 0.1$

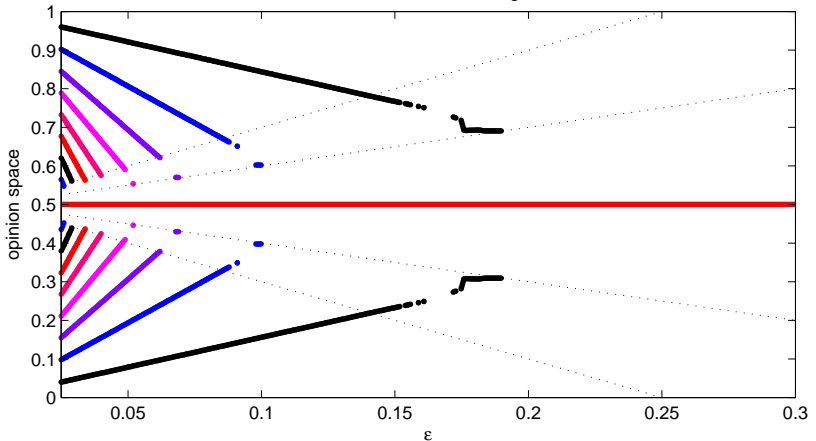


► DW model converges smoothly

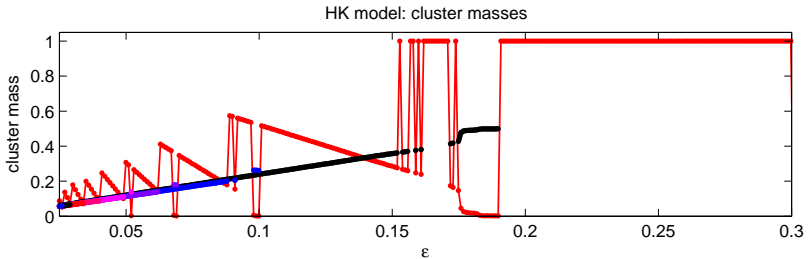
<sup>3</sup> Jan Lorenz, MSO Conference 2004, Jan Lorenz, JASSS 2006

# Cluster Location Bifurcation in Hegselmann-Krause

HK model: bifurcation diagram

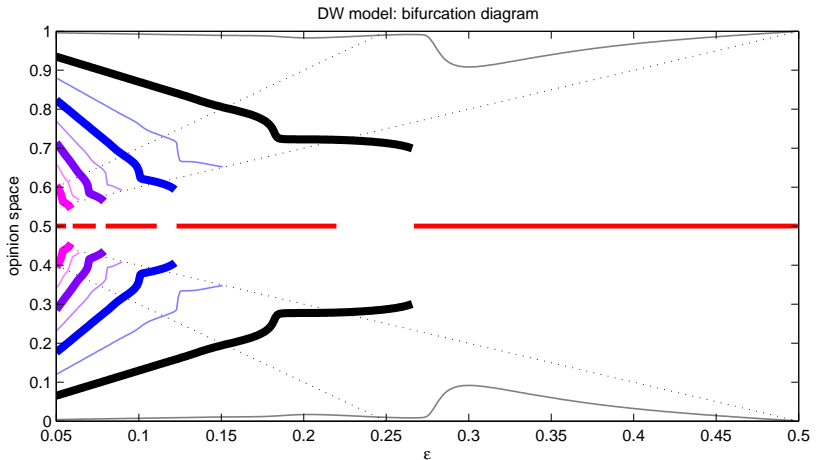


# Cluster Sizes in Hegselmann-Krause

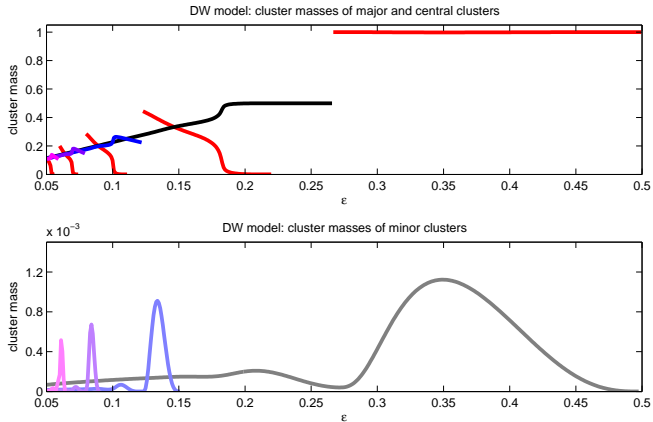




# Cluster Location Bifurcation in Deffuant-Weisbuch

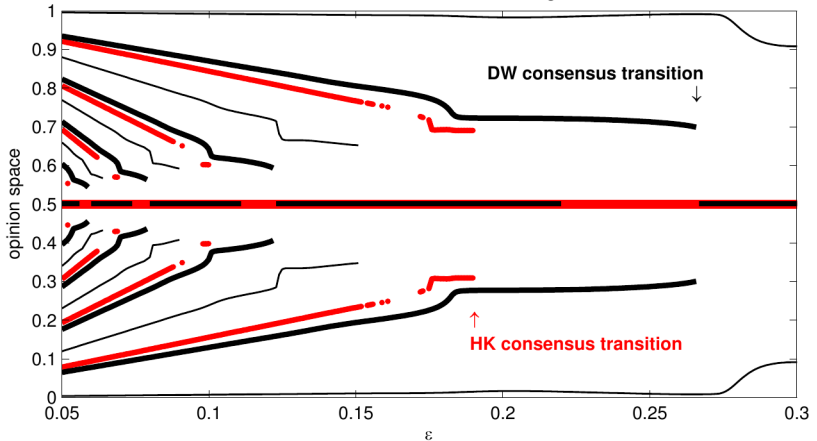


## Cluster Sizes in Deffuant-Weisbuch



# Consensus Transition

HK and DW model: bifurcation diagram



## Extensions and Simulation

- ▶ **Random Exit, Entry of new opinions** With small probability  $p$  an agent chooses a new opinion from the initial distribution
- ▶ **Heterogeneous bounds of confidence** Agents differ in their willingness to take opinions of others into account
- ▶ **Extremists** Agents with extreme opinions do not change opinion by definition
- ▶ **Higher-dimensional policy spaces** With budget restriction or not?
- ▶ **Other sources of noise** Check robustness of clustering dynamics!
- ▶ **Smooth-bounded confidence**

## Research Collaboration

Open interdisciplinary agenda for researchers with backgrounds in **many-particle physics** or **political science**

- ▶ Develop **spatial models** of **political deliberation** based on
  - ▶ bounded rationality (e.g. myopic agents)
  - ▶ incomplete information (e.g. due to limited attention or information processing capabilities)
  - ▶ heuristic strategies (e.g. for adaption, adjustment, choice of communication partners)
- ▶ Analyze **eigendynamics** of these models with
  - ▶ Monte Carlo simulation and
  - ▶ rate equations
- ▶ Far aim: Produce policy implication for communication and decision in a deliberative democracy
  - ▶ e.g. **How to foster consensus using eigendynamics?**