Fostering Consensus

in Multidimensional Continuous Opinion Dynamics under Bounded Confidence

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FRIEDRICH EBERT STIFTUNG Ph.D. Scholarship

Potentials for Complexity Science in Business, Government and the Media, Budapest, 2006

The take-away

- If you **want consensus**, bring more interrelated issues into discussion and initiate big meetings.
- If you **don't want consensus**, bring more independent issues into discussion and prevent big meetings.

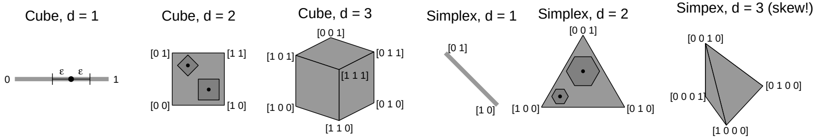
The Basics

- Continuous opinions like prices, tax rates, real numbers
- Agents tend to agree with others $\rightarrow$ averaging
    - informational reasons
    - normative reasons
- Agents tend to ignore others which differ too much in opinion $\rightarrow$ bounded confidence

Our question: **Which structural conditions foster the achievement of consensus in the agent's society?**

Simulation Setup

- $n = 200$ agents
- opinion space is a subset of $\mathbb{R}^d$, we distinguish
  - *cube* $\square^d := [0,1]^d$
  - *simplex* $\triangle^d := \{y \in \mathbb{R}^{d+1}_{\geq 0} \mid \sum_{i=1}^{n} y_i = 1\}$
- $d = 1, 2, 3$
- Distinguish distance measures p-norms with $p = 1, \infty$
  - $\left\|x^1 - x^2\right\|_1 = \sum_i \left|x_i^1 - x_i^2\right|$ *'compensators'*
  - $\left\|x^1 - x^2\right\|_\infty = \max_i \left|x_i^1 - x_i^2\right|$ *'noncompensators'*
- $\varepsilon > 0$ scales the area of confidence



Let us consider ...

- an initial opinion profile $x(0) \in (\Box^d)^n$ or $(\triangle^d)^n$
- a bound of confidence $\varepsilon > 0$
- a norm parameter $p \in \{1, \infty\}$

Repeated Meeting Process[1]

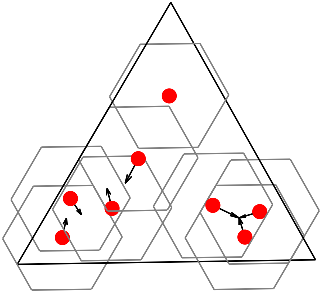
$(x(t))_{t\in\mathbb{N}}$ recursively defined through

$$x(t+1) = A(x(t),\varepsilon)x(t),$$

with $A(x,\varepsilon)$ the *confidence matrix* with

$$a_{ij}(x,\varepsilon) := \begin{cases} \frac{1}{\#I(i,x)} & \text{if } j \in I(i,x) \\ 0 & \text{otherwise,} \end{cases}$$

with $I(i,x) := \{j \mid \left\|x^i - x^j\right\|_p \le \varepsilon\}$.



one step

[1] Krause 2000, Hegselmann and Krause 2002 (JASSS)

Gossip Process [2]

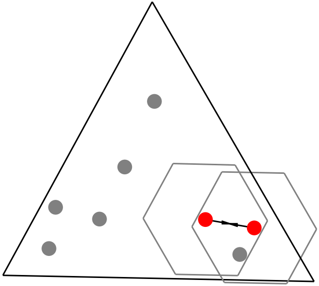
$(x(t))_{t\in\mathbb{N}}$ with in each time step $t \in \mathbb{N}$ two random agents i, j which perform

$$x^i(t+1) = x^i(t) + \frac{x^j(t) - x^i(t)}{2}$$

if $\left\|x^i(t) - x^j(t)\right\|_p \le \varepsilon$

$$x^i(t+1) = x^i(t) \text{ otherwise.}$$

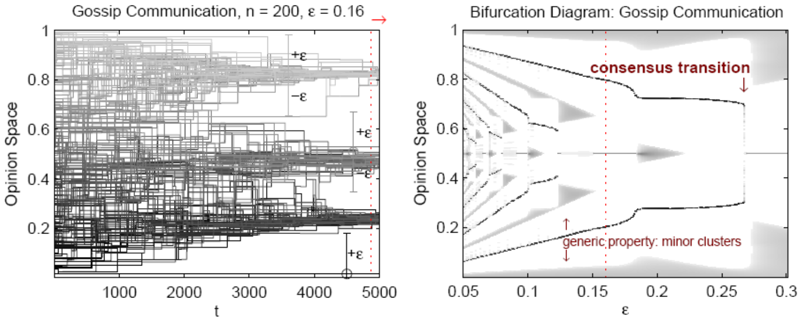
The same for $x^j(t+1)$ with i and j interchanged.



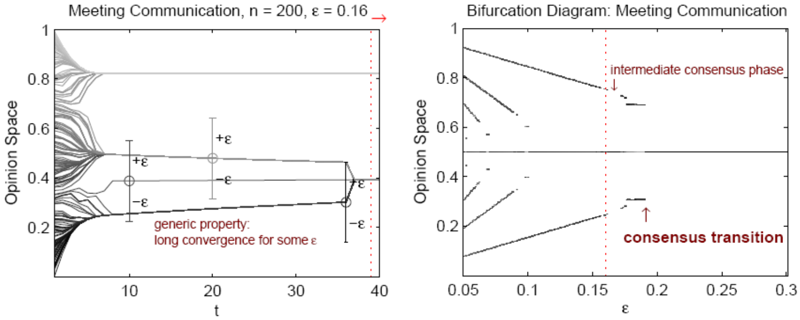
gossip-step

[2] Deffuant et al. 2000 (ACS), Weisbuch et al. 2002 (Complexity)

Gossip Dynamics

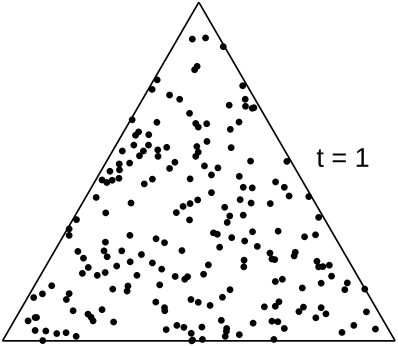


Repeated Meeting Dynamics[3]



[3] J.L., JASSS 2006

Example for $\triangle^2, \varepsilon = 0.2, p = \infty$, Repeated Meetings



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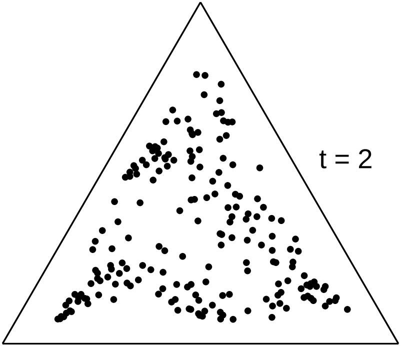

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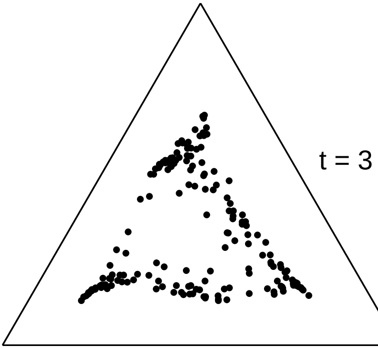

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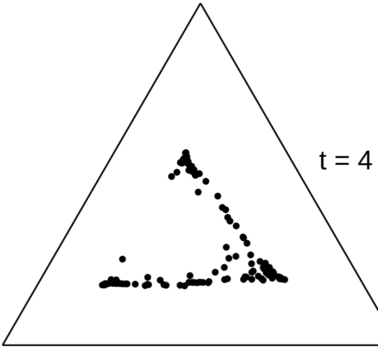

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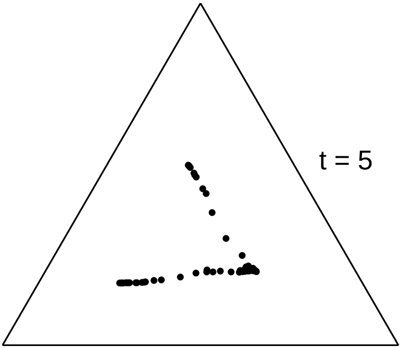

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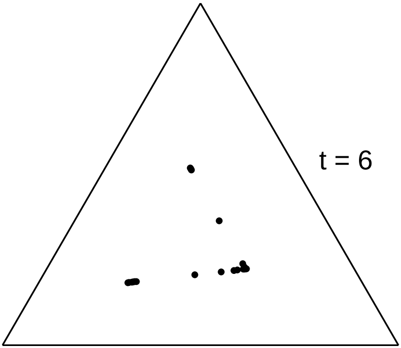

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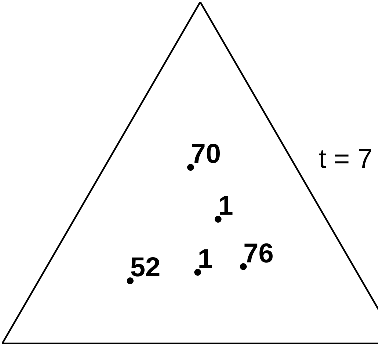

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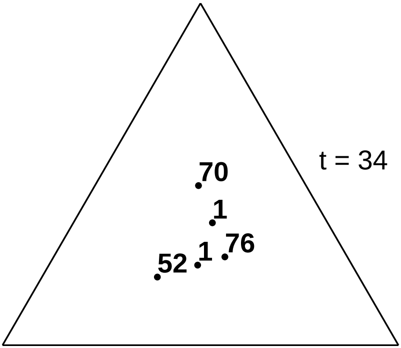

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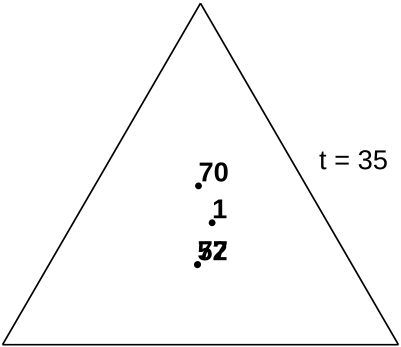

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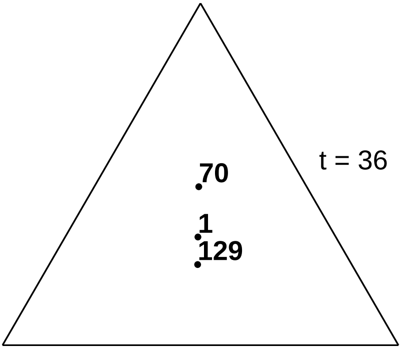

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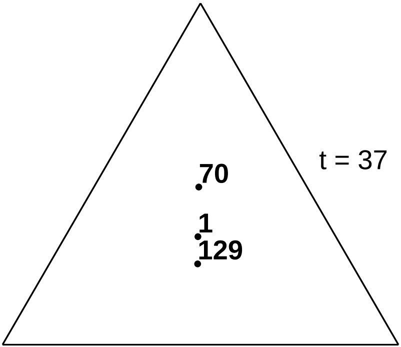

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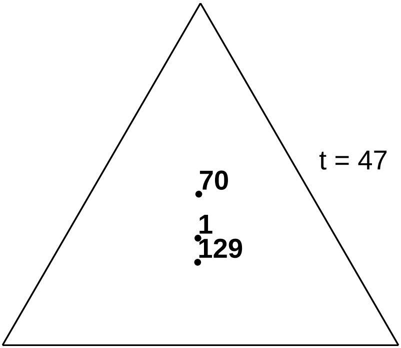

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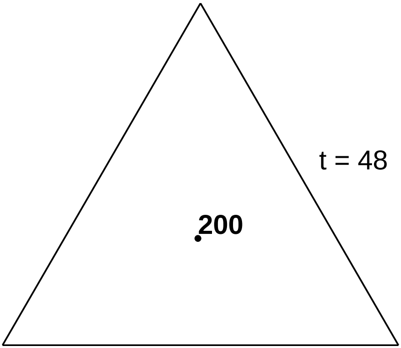
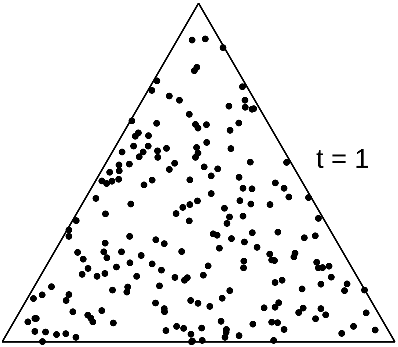


Example for $\triangle^2, \varepsilon = 0.2, p = \infty$, Gossip



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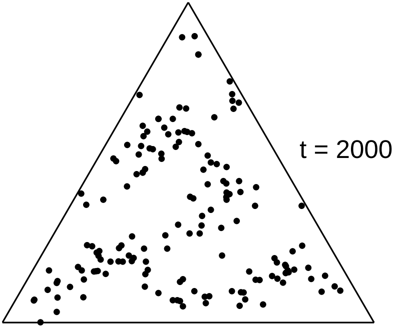

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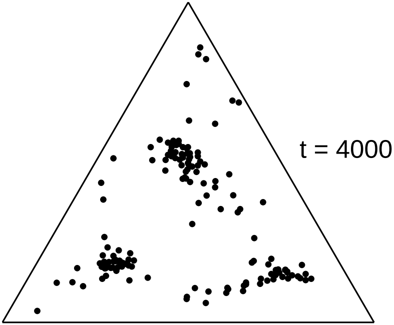

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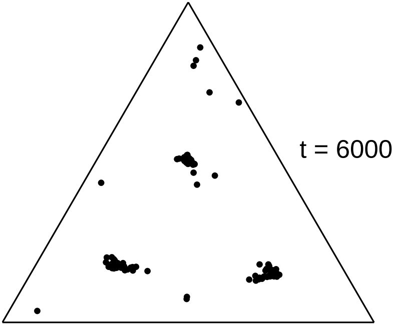

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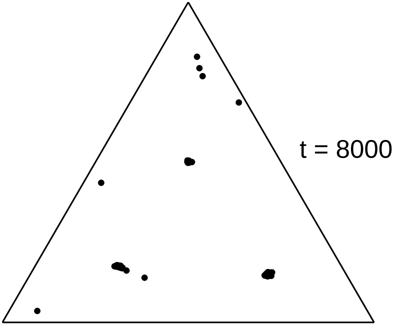

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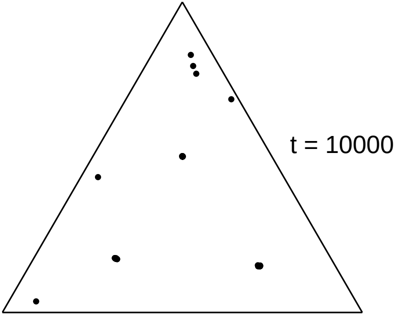

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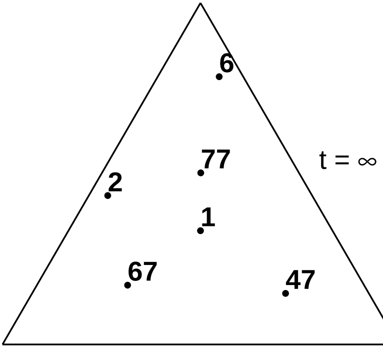
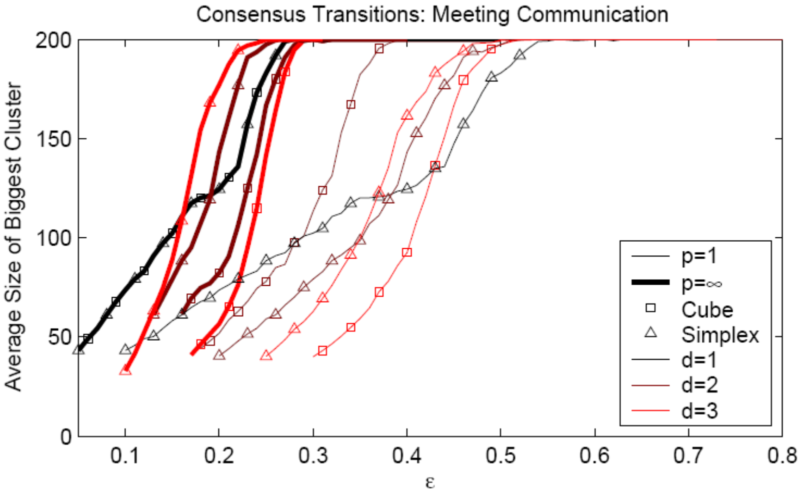


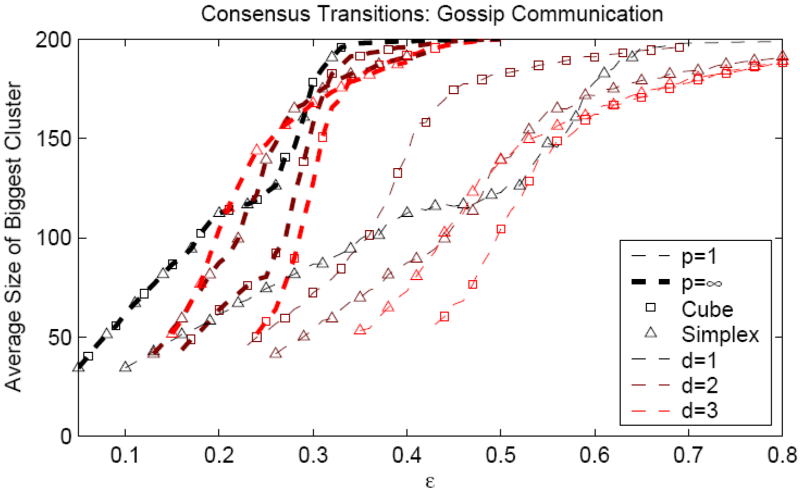
Simulation: Change of the Consensus Transition

The 'consensus measure': average size of the biggest cluster
Simulation set: 250 simulation runs vs. ε (in steps of 0.01).
Parameter space: 24 parameter settings with $\square^d$, $\triangle^d$, $d = 1, 2, 3$, $p = 1, \infty$ and meeting and gossip communication.

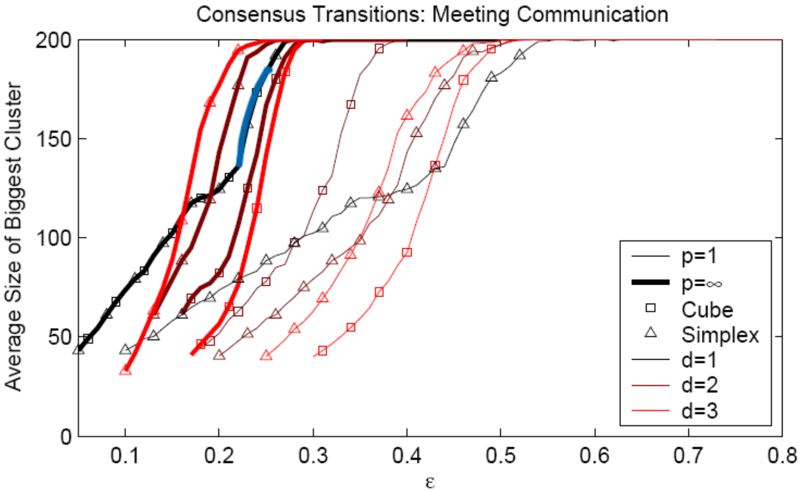
Consensus Transition for Meeting Communication



Consensus Transition for Gossip Communication



Rising *d* Fosters Consensus under Budget Constraints



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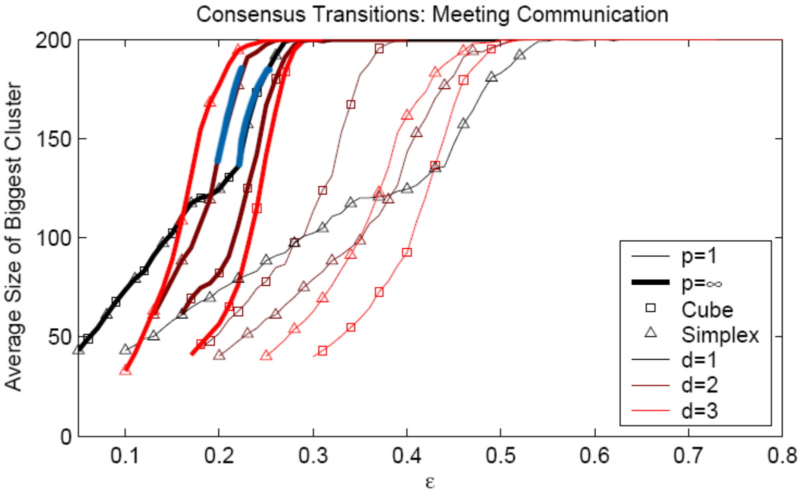

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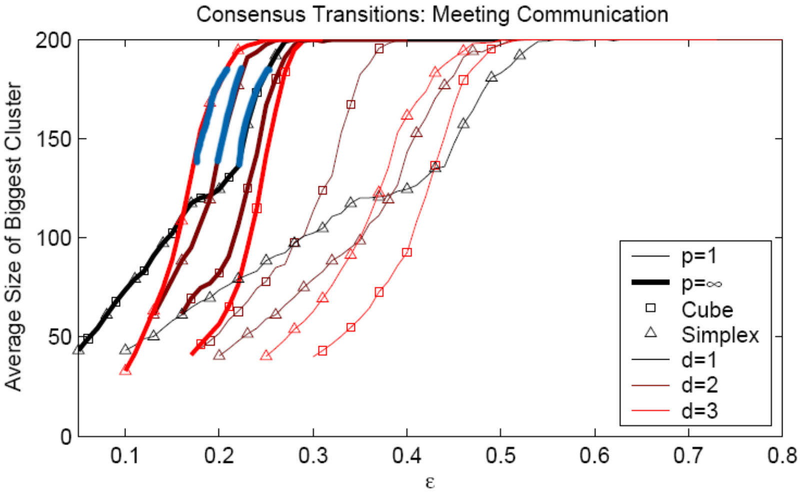
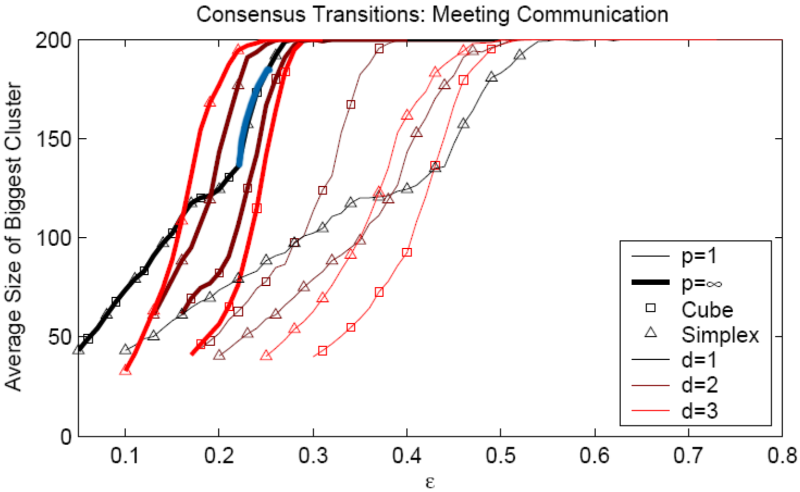


Rising *d* Weakens Consensus without



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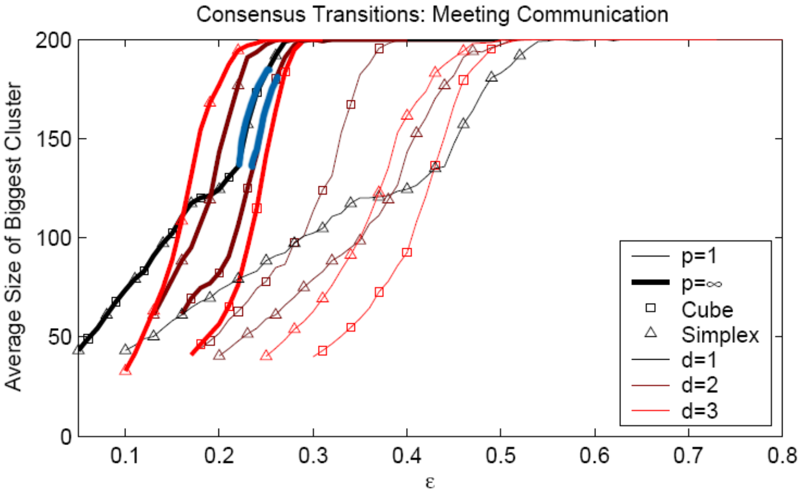

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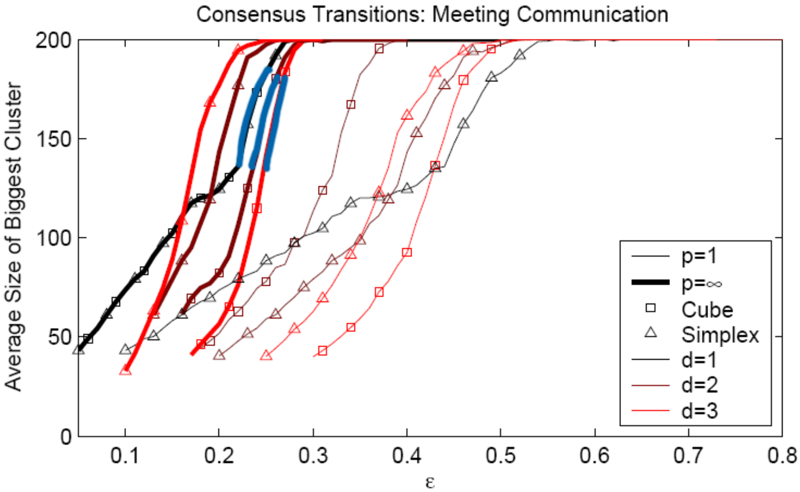
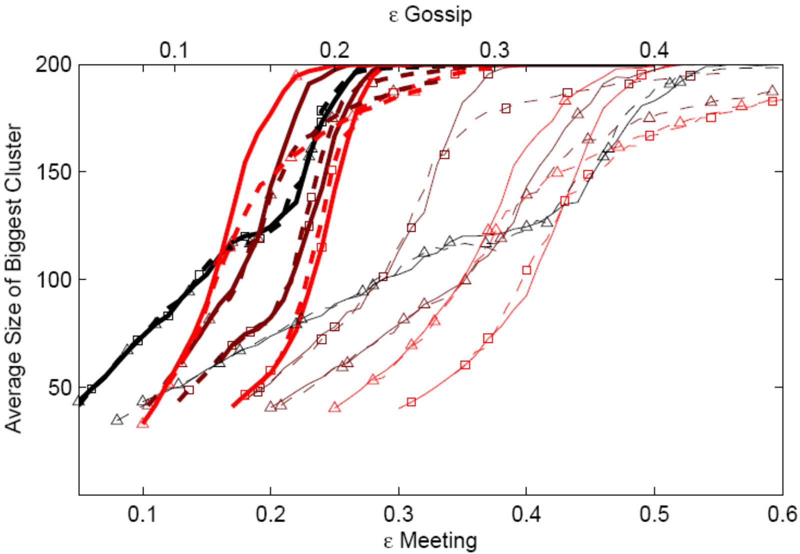


Meeting needs only 80% confidence as Gossip



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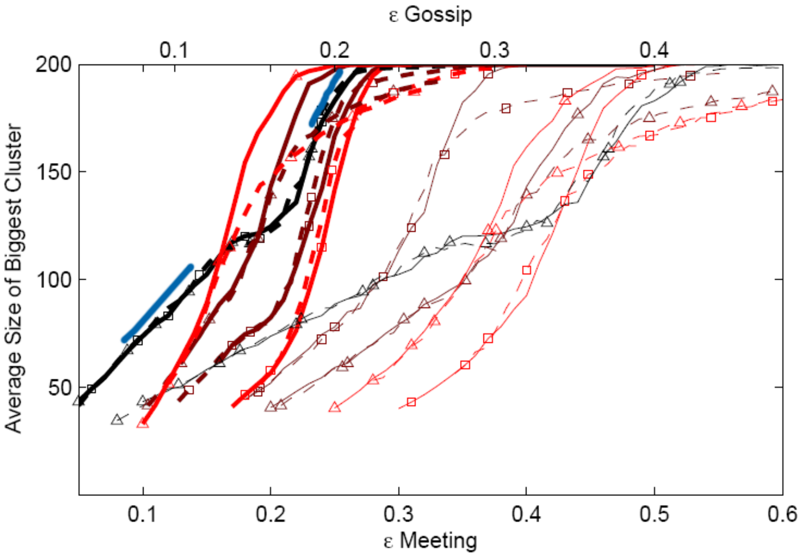

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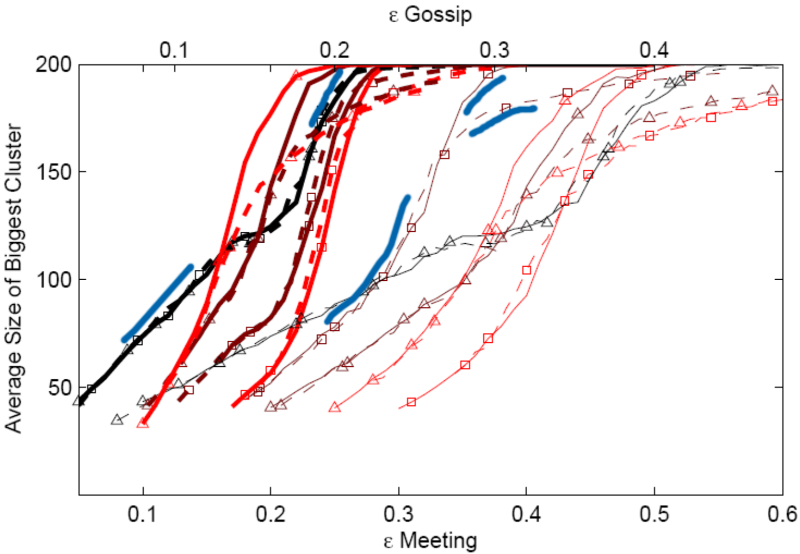

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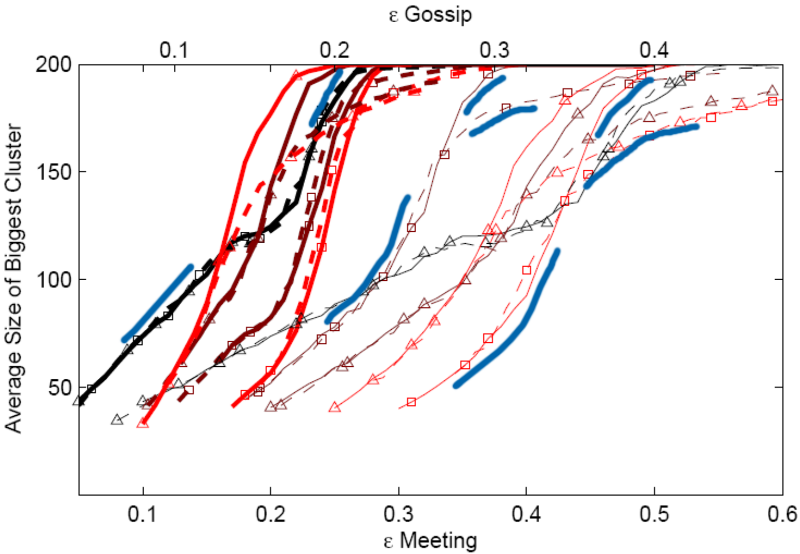

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