Continuous Opinion Dynamics: Insights through Interactive Markov Chains

Jan Lorenz

Departement of Mathematics and Computer Science
University of Bremen, Germany

Universität Bremen

FRIEDRICH EBERT STIFTUNG

August 29-31, Aruba

Outline

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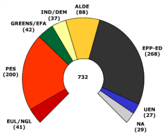
If we have continuous opinions we may compromise in the middle

Idea and Examples of continuous opinion dynamics

- We consider agents with opinions about a onedimensional issue.
- Opinion = real number.
- Agents change their opinion by compromising with others.

Examples

- political opinion left to right
- prices



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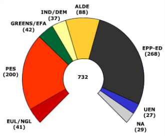

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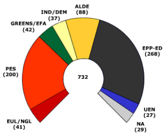

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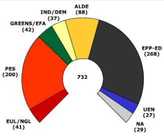

We restrict opinion dynamics by bounded confidence of our agents

Bounded confidence

- Assumption: Agents have bounded confidence
- An agent only takes an opinion into account, if it is at least ε far away from his own
- ε is called the bound of confidence
- ε is the same for all agents

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In the Weisbuch-Deffaunt ABM agents communicate pairwise

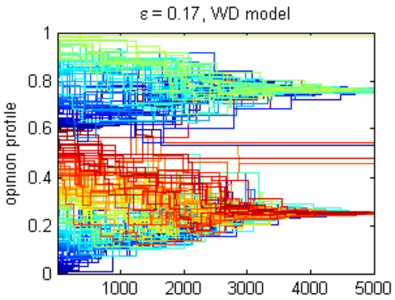
The agent-based Weisbuch-Deffuant model

Definition (WD model ABM)

Given $x(0) \in \mathbb{R}^n$, $\varepsilon \in \mathbb{R}_{>0}$ we define the random process $(x(t))_{t\in\mathbb{N}_0}$ that chooses in each time step $t \in \mathbb{N}_0$ two random agents $i, j \in \underline{n}$ which perform

$$x_i(t+1) = \frac{x_i(t) + x_j(t)}{2}$$

if $|x_i(t) - x_j(t)| \leq \varepsilon$. The same for $x_j(t+1)$.



In the Hegselmann-Krause ABM each agent averages all opinions he trusts

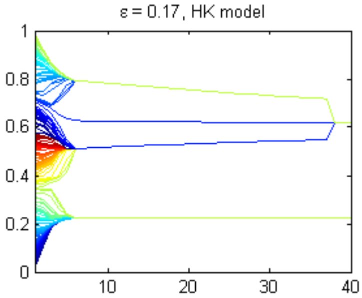
The agent-based Hegselmann-Krause model

Definition (HK model ABM)

Given $x(0) \in \mathbb{R}^n, \varepsilon \in \mathbb{R}_{>0}$ we define the *HK process of opinion dynamics* through $x(t+1) = A(x(t))x(t)$ with *confidence matrix*

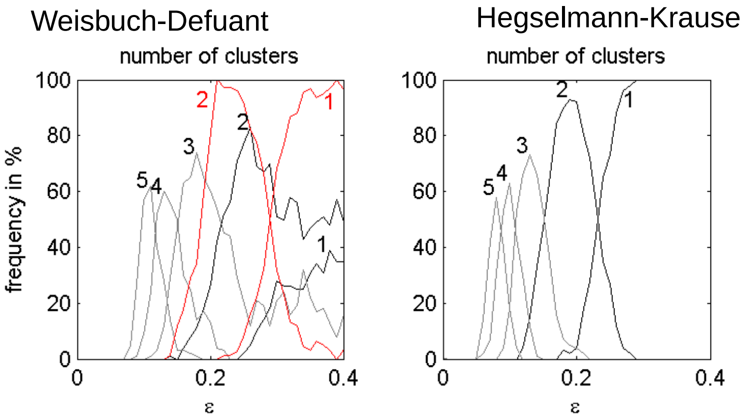
$$a_{ij}(x) := \begin{cases} \frac{1}{\#I(i,x)} & \text{if } j \in I(i,x) \\ 0 & \text{otherwise,} \end{cases}$$

$$I(i,x) := \{j \in \underline{n} \mid |x_i - x_j| \leq \varepsilon\}.$$



While the WD model may produce outliers, the HK model may lead to meta-stable states

Comparison of the WD and HK ABM



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In the WD interactive Markov chain we switch to infinite agents and finite opinion classes

The Weisbuch-Deffuant interactive Markov chain

Definition (WD transition matrix)

Given $p \in S^{1\times n}, k \in \mathbb{N}$ the *WD transition matrix* is

$$b_{ij} := \begin{cases} \frac{\pi^i_{2j-i-1}}{2} + \pi^i_{2j-i} + \frac{\pi^i_{2j-i+1}}{2} \\ q_i, \quad \text{if } i = j, \end{cases}$$

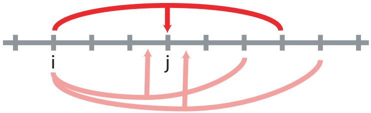
with $q_i = 1 - \sum_{j\neq i, j=1}^{n} b(p,k)_{ij}$

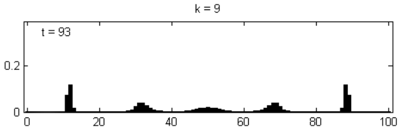
$$\pi^i_l := \begin{cases} p_l, & \text{if } |i-l| \leq k \\ 0, & \text{otherwise.} \end{cases}$$

Definition (WD IMC)

Given $p(0)$ the *WD IMC* is

$$p(t+1) = p(t)B(p(t),k).$$





Similar for the HK interactive Markov chain

The Hegselmann-Krause interactive Markov Chain

Definition (HK transition matrix)

Given $p \in S^{1\times n}, k \in \mathbb{N}$ the *HK transition matrix* is

$$b_{ij} := \begin{cases} 1 & \text{if } j = M_i, \\ \lceil M_i \rceil - M_i & \text{if } j = \lfloor M_i \rfloor, \\ M_i - \lfloor M_i \rfloor & \text{if } j = \lceil M_i \rceil, \\ 0 & \text{otherwise.} \end{cases}$$

$$M_i := \sum_{|i-m|\leq k} m p_m \Big/ \sum_{|i-m|\leq k} p_m .$$

is the *k-local mean*.

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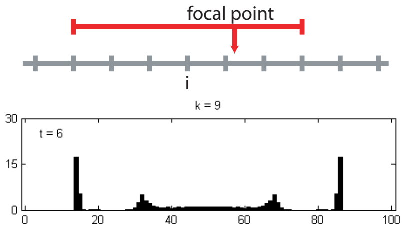
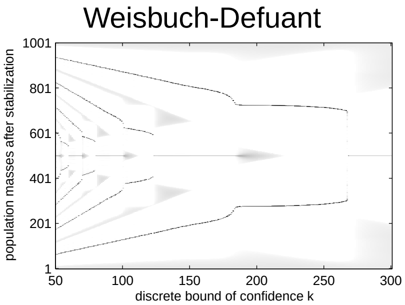
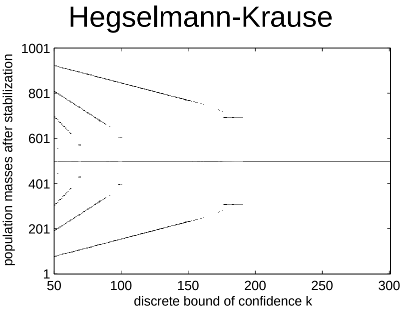


Outliers in the WD model and meta-stable states in the HK model appear more drastic in the IMCs

Comparison of the WD and HK IMC

Bifurcation diagrams





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What we should take home

- Continuous opinion dynamics under bounded confidence leads to interesting clustering phenomena regarding the communication regime. E.g. minorities and metastability
- Interactive Markov chains are a tool to capture the underlying dynamics for many agents in one bifurcation diagram
- Differences between WD and HK appear more drastic, especially the 'consensus strikes back' phase

- Attention! Modelling results should be retransferred to reality only in a qualitative manner

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References I

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References II

Jan Lorenz
Further publications and working papers on opinion dynamics, 2003-2005
www.janlo.de

Thank you for your attention!