Convergence to Consensus in Multiagent Systems

Setup: N agents, agent $i \in \{1,\dots,N\}$ has initial opinion $x_i(1) \in \mathbb{R}$, $x(1) \in \mathbb{R}^N$ is initial opinion profile

Question: Do agents converge to a consensus when repeatedly confronted with the opinions of others?

Mathematical Conditions

Behavioral Model Agent i gets information of some other agents' opinions and takes a **weighted average**. Network topology of information exchange and weighting may change over time or may be a function of the opinion landscape. Implies systems dynamics as:

$$x(t+1) = A(t)x(t) = A(t)\cdots A(2)A(1)x(1)$$

with $A(t)$ row-stochastic (implements weighted averaging).

Mathematical Questions

1. $\lim_{t\to\infty} A(t)\cdots A(2)A(1) = A(\infty,1)$ exists?
2. $A(\infty,1)$ consensus matrix (all rows equal)?

Results

Result 0. When (I) holds there is $0 < t_1 < t_2 < \dots$ such that $A(t_s-1)\cdots A(t_{s-1}) =: A(t_s,t_{s-1})$ have **same zero pattern**

$$\dots \simeq A(t_s,t_{s-1}) \simeq \dots \simeq A(t_3,t_2) \simeq A(t_2,t_1).$$

Conditions

(I) **Self-confidence** $A(t)$ has positive diagonal

(II) **Positive weights uniformly bounded** from below. There is $\delta > 0$ such that for all t, $\min_{\{ij \mid a_{ij}(t)>0\}} a_{ij}(t) > \delta$

(III) **Mutual confidence** $A(t) \simeq A(t)^T$

(IV) **Bounded lengths of intercommunication intervals.** There are $T_1, T_2 > 0$ such that for all s it holds, $t_s - t_{s-1} < T_1 + T_2 \log(s)$ and $T_2 \le -\frac{1}{\log \delta}$. (Slow growth allowed!)

(V) $A(t_s,t_{s-1})$ has **only one connected cluster without outlinks**.

Result 1. Opinions converge

- if (I), (II), (III); or
- only in connected clusters without outlinks, if (I), (II), (IV).

Result 2. Opinions converge to consensus

- if (I), (II), (III), (V), or
- if (I), (II), (IV), (V).

Consider $A(t)$ is a function of $x(t)$? Conditions (I), (II), (III) might be easy to check for the function, but (IV) and (V) are conditions on accumulations of matrices of unknown length and thus very difficult to check!

References

Lorenz, J. **Repeated Averaging and Bounded Confidence** (2007) Doctoral Thesis, Universität Bremen

Lorenz, J. **A stabilization theorem for dynamics of continuous opinions** (2005) Physica A, 355(1), 217–223

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Human Behavior

Experimental Setup: $N = 12$ humans in a computer laboratory
12 experimental sessions, **6 estimation tasks**, e.g.
"How many murders were registered in Switzerland 2006?"
5 rounds of estimation (truth unknown to agents)
Payment for each estimate when low deviation to truth
3 conditions of information propagation after each round:

(i) **no information** answer five times the same question

(ii) **aggregated information** receive the arithmetic mean of all 12 estimates

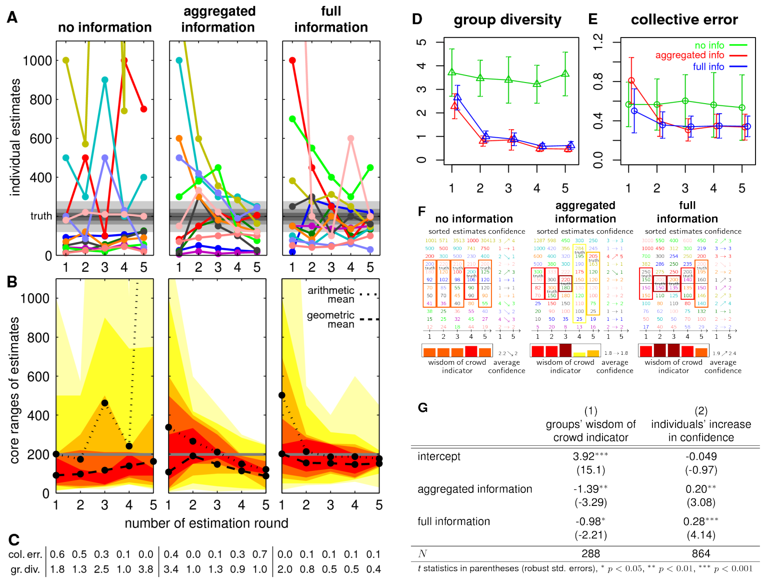
(iii) **full information** receive graphics of trajectories and list of all estimates

±10% → 4 points
±20% → 2 points
±40% → 1 point
else → 0 points

Experimental Questions

1. Do agents converge through information feedback?
2. **Does information feedback has an impact on the wisdom of crowds effect?**

Results



Result 1. Convergence but **not towards a full consensus**.
Reason: Agents belief in "Others also don't know." this implies some random fluctuation as under the "no info" condition.

Result 2. Impact of Information Feedback:

Social Influence Effect Diversity of estimates diminishes without lowering collective error. (A,B,C; Stats: D,E)

Range Reduction Effect Position of truth drops out of range of estimates (or to periphery). (F; Stats: G)

Confidence Effect Agents become more confident. Danger of extrapolation to crowd! (F; Stats: G)

Reference

J. Lorenz, H. Rauhut, F. Schweitzer, D. Helbing. **How social influence can undermine the wisdom of crowd effect** (May 2011) PNAS