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Designing Participatory Budgeting

Mathematics of opinion dynamics and aggregation

Jan Lorenz | Universität Bremen | post@janlo.de
Frederico Menino | Universidade de São Paulo | fremenino@hotmail.com

In this paper, participatory budgeting (especially in Porto Alegre) is seen as a **formal mathematical model** of **opinion dynamics** and **social-choice**. We relate it to the classical social-choice problem mixed with the ideas of the wisdom of crowds and **social consensus**. Mathematical Simulation of opinion dynamics are done to analyse the **impact of the mechanism** on the finding of a social consensus.

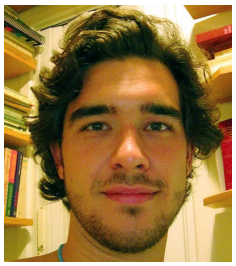
With these mathematical results, we aim to contribute to the debate about introducing participatory budgeting in Germany and how to extend it in Brazil.

The simulation of opinion dynamics leads to the result that giving the crowd **more categories** to distribute money to has an impact that **fosters** at least a vast majority of the group to find a social consensus. But this effect slows down so that about eight categories seem to be enough.

We conclude by putting our results into the general picture covering the **political barriers** of implementing participatory budgeting: representation, power balance and institutionalization. And we add some remarks about participatory budgeting as a tool to evolve from technobureaucracy to **technodemocracy** and about the civil society as a forth force in democracy.

Jan Lorenz (29)

- has studied mathematics and philosophy at the University of Bremen
- is working on his Ph.D. thesis about "Modelling, analysis and simulation of opinion dynamics"
- this study about participatory budgets was his first contact to Brazil
- is generally interested in communicating and discussing results of social simulation, sociophysics to policy makers



Frederico Menino B. de Oliveira (23)

- last year student in Social Sciences at the University of São Paulo (Brazil)
- former special student at JFK School of Government (Harvard University)
- in 2004 he has concluded a two-year undergraduate research on Participatory Budgeting in Brazil (Democracia e Governo Local: uma revisão dos Estudos sobre o Orçamento Participativo), sponsored by CNPq (National Research Funding)

1. Introduction

A participatory budget is a budget for a commune or city which has been developed under participation and decision of the citizens.
The flagship of participatory budgeting is the southern Brazilian city of Porto Alegre, where the orçamento participativo (participatory budget) has been established 1989 with growing success (see Boxes 1 and 3). The idea has obtained worldwide publicity and spread all over the world. In Europe where lack of participation of citizen is often bemoaned the idea become popular under the headline "Learning from the south" (for the German initiatives see Box 2, for a review of European initiatives Allegreti and Herzberg (2004)).

Box 1. Participatory Budgeting in Brazil: Origins and history

The origin of participatory budgeting in 1989 can be traced back to a long process of social mobilization in Brazil. Only as the result of singular political and social dispositions, OP could be idealized and first implemented in Porto Alegre, by the Workers' Party (PT).

As stated by Carvalho (2003), Brazil has watched, during the 1990's, the generalization of a "participation discourse", which emerged from the democratization process occurred in the previous decade. This process, which culminated in the Democratic Constitution of 1988, was led by a strong alliance between anti-authoritarian shares of the liberal elite and socialist organizations which had been strengthened since the late 1970's. Among these were reinforced unions, recently created social and local movements – as the Landless Movement (MST) –, and new leftist parties, such as the Workers' Party. However, more than just freeing Brazil from the Military Regime, those organizations aimed to revert the authoritarian and paternalist tradition of the country by enhancing direct participation of civil society in public issues.

Besides this sociopolitical background, OP could also be implemented by significant changes, imposed through the Constitution, in the distribution of revenues among States and Municipalities. After 1988 the amount of federal transference to the municipalities increased 35% with the creation of the Municipal Participation Fund (FPM).

All these factors were decisive in Porto Alegre's case. Although the city was passing through a difficult financial period by the time OP was installed, the city budget has considerably increased since 1989 (Silva, 2001), allowing Olivio Dutra (P.Alegre's mayor from 1989 to 1992) to promote a stable plan of social and fiscal reforms based on popular participation. Moreover, PT's historical approval in the State of Rio Grande do Sul, added to the power of well articulated worker unions and neighbourhood associations, contributed to the irrefutable success of the program (Utzig, 1996).

The model of Porto Alegre has been awarded at the conference UN-Habitat II 1996.

From a more abstract point of view the process of participatory budgeting aggregates a budget plan out of the demands and desires of the population of the city. Under this point of view it seems possible to describe the process as a **formal mathematical model**.

From that mathematical point of view we want to contribute to the question what aims can be achieved by participatory budgeting. We focus on two problems: participatory budgets as a solution to the social choice problem and participatory budgeting as a process to foster social consensus.

In all processes of participatory budgeting there comes a point where citizens or districts have to propose a list of priorities which has to be aggregated to a common priority list of the local authority. To manage this task is known as the problem of **social choice**. The aggregation has to be done by a certain voting system. The mathematical analysis of voting systems has proved the fact that no voting system is perfect and one has to choose very carefully which one fits best to the issue. We argue that a voting system, where each voter proposes a distribution of percentages to each alternative such that all percentages sum up to 100%, fits to some extend better to the aims of aggregation in participatory budgeting. Nevertheless, one has to take care of the potential of tactical voting.

The idea that participatory budgets foster **social consensus** in times of short cash has been claimed in the Berlin project (bpb 2004) as one of the ten golden arguments for participatory budgeting.

Mathematics can contribute to this issue by simulation of **opinion dynamics** with a big number of citizens each having a desired distribution of money to the alternatives as an opinion.

Box 2. Participatory Budgeting in Germany: At the very beginning

In Germany all local authorities who implemented a participatory budget are very young. In the project "Kommunaler Bürgerhaushalt" in North-Rhine Westphalia, six local authorities were supported to launch participatory budgets in regions with 21.000-181.000 inhabitants and different city budget problems (from balanced to heavy deficit).

The project "Bürgerhaushalt in Großstädten" (started 2003) has Berlin as the main target to develop and implement a participatory budget.

In contrast to the social choice view the opinion dynamics view does not regard aggregation of different opinions to one opinion but focuses on the possibility that citizen may change their opinion if they hear the opinions of others. If this exchanging and revising of opinions is repeated the citizens may come to a common agreement without aggregation. This is what we call social consensus in our formal model. In our formal model we will postulate that citizens are only revising their own opinion by taking into account other opinions that are not too far away from their own opinion.

We argue by simulation that, global meetings in a process of opinion dynamics have a stronger impact on finding a consensus than relying on one-to-one gossip communication. And surprisingly, the number of alternatives where money can be distributed to has a strong impact, too. More alternatives foster the finding of a consensus.

Box 3. General Information and Basic Data

According to Grazia (2002) , there had been 103 OP experiences in Brazil until 2000 – 52 of these implemented by PT administrations. The author also predicted that other 37 experiences would be inaugurated in Brazil until 2003. Furthermore, Grazia noted only 8 cities where the local government has actually abandoned OP until 2002.

The total amounts of revenues vary from city to city, but according to most accounts the money available for public deliberation usually ranges from 2 to 6% of the total municipal revenues. In Porto Alegre – a very atypical example – 7.5% of the city budget was spent in projects directly voted by the population during the first 4 years of the program. This rate decreased to 3.5% in 1994, affecting the popularity of OP among citizens (Ricci, 2003).

In relation to participation, most OP experiences tended to increase their rate of attendance across the years. The numbers for Porto Alegre follow the same pattern: while in 1990 only 348 individuals attended the first round meetings, in 2005 they were 10.458. Marcelo Silva (2001), who analyzed OP in Porto Alegre and a number of other cities, observes that there is also a tendency for higher rates of participation among poorer citizens. The wages of more than 55% of those who attended the regional assemblies in 2000 were lower than R$800,00. However this number represents a decrease in relation to previous years, showing that the middle class is participating more nowadays.

The OP has lead to an increase of the quality of infrastructure in Porto Alegre, e.g. households with sewage disposal from 46 % (1989) to 84 % (1999). However, some authors (e.g. Rizzotti, 1999) argue that other mayors – like Jaime Lerner (PMDB) in Curitiba – developed totally different social agendas and succeeded as much as the PT government in Porto Alegre. Participation, therefore, would not be the axis of PT's success in the South; the deeper reason for this was the significant raise in the local revenue.

One disclaimer in advance: Mathematical modelling of social systems should not be compared with mathematical modelling of the weather or climate. While their aim is to capture all laws of nature to forecast the future out of a given situation we try to capture only very selected aspects we regard as relevant to discover driving forces of dynamical behaviour. The aim is to get some rational based hints how to design and steer opinion dynamics to stimulate good outcomes, in our case consensus.

The paper is organized as follows. In the next section we will describe the participatory budgeting process in Porto Alegre, where we will point out at which time social choice and opinion dynamics happens. Sections "The social choice problem" and "Opinion dynamics" are guided from the mathematical point of view. In the second last section we address briefly some challenges that arise frequently while implementing and realising participatory budgets and link them to the mathematical analysis. The last section concludes and puts some aspects into a broader picture.

2. The process of Orçamento Participativo in Porto Alegre

The process of Orçamento Participativo (OP) has several components but must be understood as a whole. It is a combination of a delegating system, a voting system to aggregate local projects and priorities and a time schedule of giving account, leaving space for opinion dynamics and requiring of decisions.

The scope of the participatory budget in Porto Alegre is only the investment budget not the yearly obligations for salaries, support of infrastructure and pensions. The OP has been implemented additional to the existing entities of representative democracy (see figure 1). We try to formulate the process as pure as possible so that we can see it as an abstract process.

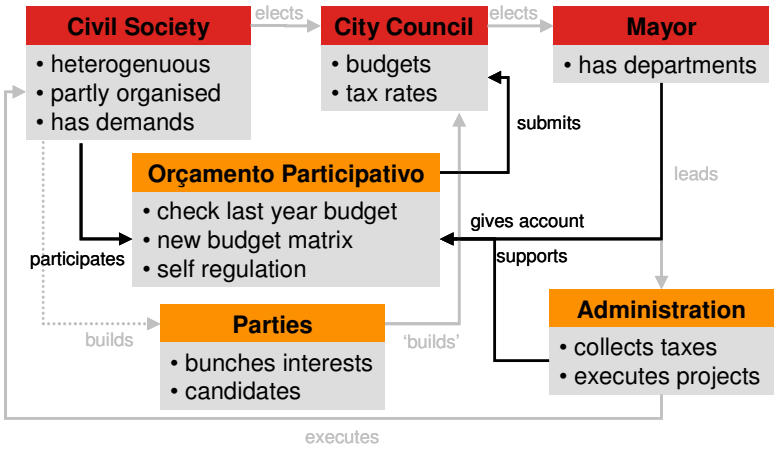


Figure 1. Implementation of the OP

2.1 Delegation structure

The city of Porto Alegre has been divided into 16 districts. The guideline of the division was to find districts that have a kind of social cohesion felt by the population additional to the geographical cohesion. Thus, the districts differ in size of population.

The administration divides the public demands into 14 categories (e.g. Habitation, Education, Health, Asphalting …). Each demanded project of the citizen has to be assigned to one category. The categories are more or less connected to one department of the administration.

In general the citizens were asked in the Orçamento Participativo about concrete demands for city investments in infrastructure or special projects in their district or city wide. But they were also asked about priorities of the categories. To get this process done the citizens elect a forum of delegates that debates and balances the local projects and priorities between citizens and local initiatives. One mathematical tool they are suggested to use to aggregate different priorities of micro-regions (e.g. neighborhoods) is to collect priority lists of each neighborhood and give points to their priorities (1^{st} 14 points, 2^{nd} 13 points, …, 14^{th} 1 point). Adding this points up and sorting gives a district wide priority list.

Additionally, the citizens of each district elect two members for the OP-council which takes over if all project proposals and category priorities of the districts were made. This council works out the city-wide budget plan under consideration

of the local priorities and further technical criteria (population and lack of infrastructure of each district).

To get the election of the delegates done and for regional discussions and the decision about the priorities there is one official meeting held in each district.
Additional to the district meetings there are six thematic meetings which are organized in the same way, but they address and prioritise issues related to their topic for the whole city.

Each meeting elects a fixed number of delegates which is determined by the number of participants of the meeting (in 2005 5 %). Figure 2 shows how delegates are elected.

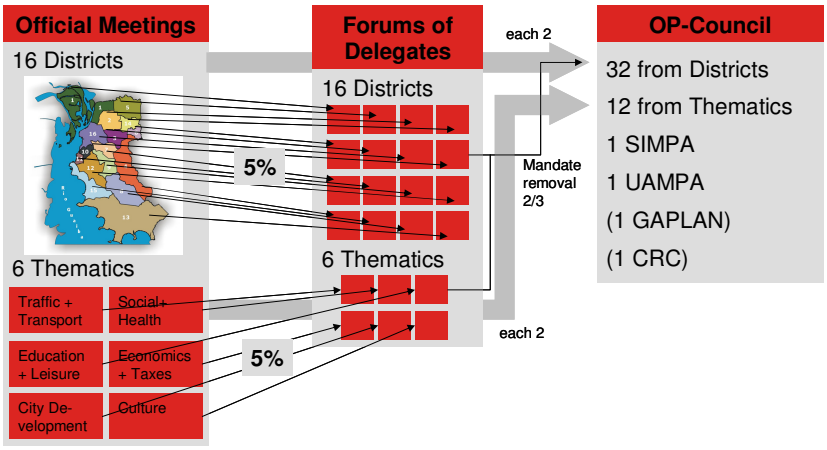


Figure 2. Delegation in the OP.

2.2 The evolution of the investment plan

The point where the OP-council takes over from the forums of delegates is a city wide meeting, where they discuss city-wide topics. After this meeting the OP-Council starts with the discussion of the budget plan proposal.

The budget plan contains (roughly) only the distribution of money to the categories in each district and not the concrete projects. This budget plan is submitted to the city council which must decide on it (or on another changed budget plan). The fact that the city council does not decide on specific projetcts goes back to Brazilian tradition and is in contrast to Germany. This fact makes it difficult for the city council to find reliable arguments against the proposal of the OP council. Thus, usually the city council approves the budget plan only with minor changes.
After the decision of the budget plan the OP-council debates with the forums of delegates in each district the investment plan. It should contain all projects that will be realized within the given budget plan (see figure 3).

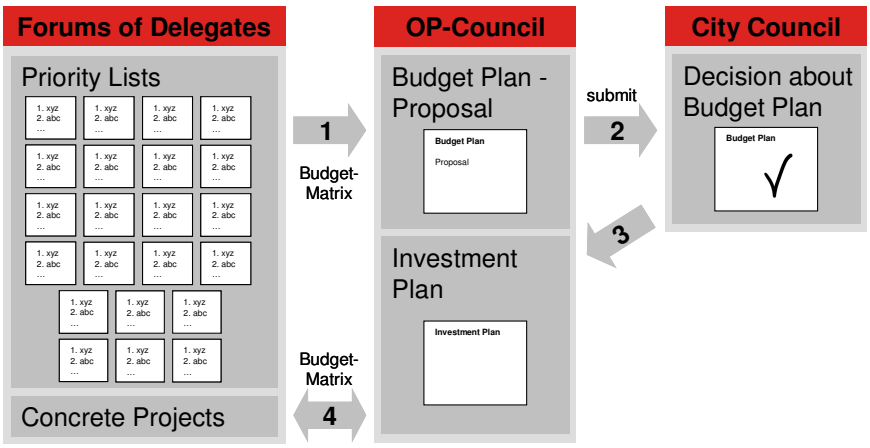


Figure 3. The evolution of the investment plan

2.3 Allocation of money with the priority matrix

The OP council has to respect several rules and a mathematical framework (Regimento interno, see references) when it allocates the funds. The heart of these rules is a matrix which contains the priorities of all 16 districts. This **priority matrix** contains in each row points from one district to the most demanded categories. The points are 4 for the top priority, 3 for the second and 2 and 1 for the third and forth priority. (See table 1 for an example. Gray number display points for the full ranking of the districts.)

Table 1. Priority matrix of the OP in Porto Alegre 2005

District \ Category	01 Underground Engeneering	02 Water and Sewage	03 Habitation	04 Asphalting	05 Education	06 Social Assistence	07 Health	08 Transport and Traffic	09 Areas of Leisure	10 Sports and Leisure	11 Street Lightning	12 Eco. Dev., Tax, Tourism	13 Culture	14 Environmental Rehab.
01 HUMAITÁ, ...	3	8	4 13	6	3 12	2 11	9	1	2	5	4	1 10	7	0
02 NOROESTE	5	2 11	4 13	1	9	8	3 12	3	2	7	4	1 10	6	0
03 LESTE	5	6	4 13	7	3 12	2 11	1 10	4	1	3	2	9	8	0
04 LOMBA DO PIN.	7	8	1 10	4 13	2 11	9	3 12	5	4	2	6	3	1	0
05 NORTE	9	3 12	4 13	8	2 11	7	1 10	2	0	6	4	5	3	1
06 NORDESTE	5	2 11	4 13	1	9	8	3 12	3	2	7	4	1 10	6	0
07 PARTENON	5	8	4 13	2 11	3 12	1 10	9	4	3	1	2	6	7	0
08 RESTINGA	2	6	2 11	9	4 13	3 12	8	3	4	5	1	1 10	7	0
09 GLÓRIA	9	8	3 12	7	4 13	1 10	2 11	3	1	5	2	0	4	6
10 CRUZEIRO	6	7	4 13	9	3 12	1 10	2 11	3	0	4	2	5	8	1
11 CRISTAL	5	9	4 13	6	7	8	1 10	3	0	2	1	3 12	2 11	4
12 CENTRO-SUL	6	7	4 13	1 10	3 12	2 11	9	2	4	8	1	5	3	0
13 EXTREMO-SUL	4	5	4 13	1 10	2 11	9	3 12	7	2	6	1	3	8	0
14 EIXO-BALTAZAR	5	9	4 13	8	3 12	1 10	2 11	4	1	6	3	7	2	0
15 SUL	1 10	9	2 11	3 12	4 13	7	8	5	4	6	3	1	0	2
16 CENTRO	2	7	4 13	6	3 12	1 10	8	4	3	5	1	2 11	9	0
TOTAL	**1** 88	**7** 131	**56** 200	**11** 124	**39** 181	**14** 151	**21** 162	**0** 56	**0** 33	**0** 78	**0** 48	**9** 107	**2** 90	**0** 14

At first, the OP council computes the total number of points for each category from the priority matrix. This list gives the council a guideline for the overall city wide demands. With this background they construct the budget plan proposal.
After the acceptance of the budget plan the OP council has a fixed amount of money to spend in each category. A part of the money in each category is distributed on some city wide investments which where proposed from the thematics. Another part is to be redistributed to the districts. For this redistribution the OP council has to respect three technical criteria: the number of inhabitants of each district (1 to 4 points, general weight 2), the lack of infrastructure (1 to 4 points, general weight 4) of each district and the priority list of each district (according to the priority matrix, general weight 5).

The best example is the category “Asphalting”: For each district the OP council computes the points for the region in each of the three technical criteria multiplied by the general weights. The percentage of points a district gets at the total sum of all points is the percentage of the total number of kilometers (assigned in the budget plan) to asphalt in this district.

The whole computation is called the **budget matrix**. This method is some times called **‘arithmetic of equitable democracy’**, e.g. by studies of the World Bank.

2.4 Support and time schedule

During all these activities the citizens working on volunteer base in the OP were supported by certain departments of the city’s administration. The CRC (Coordenação das Relações com a Comunidade – Coordination of Public Relation) is responsible for the organization of the official meetings. The mayor itself attends every district and thematic meeting. The GAPLAN (Gabinete de Planejamento – Planning Department) is a central planning institution which coordinates the implementation of demands and priorities between the several departments of the administration (see figure 4).

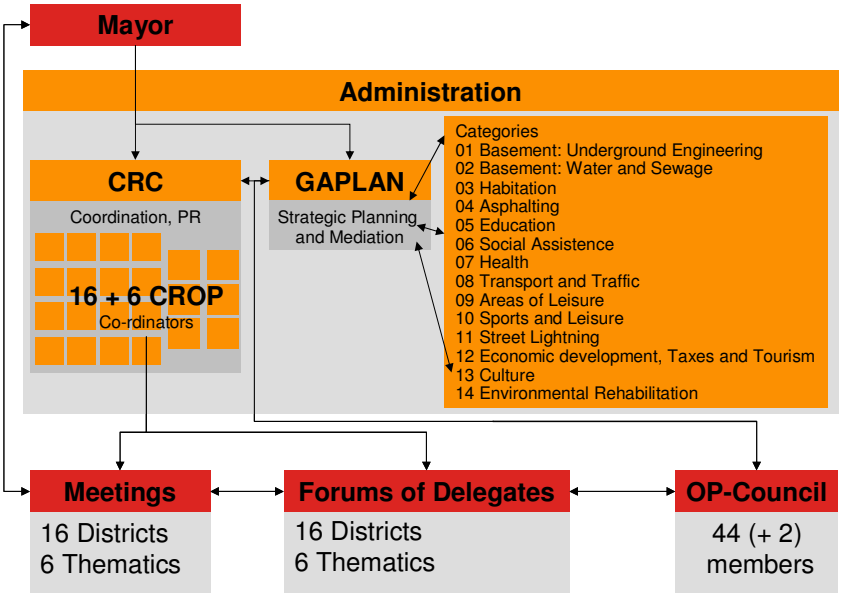


Figure 4. Departments of the city’s administration involved in the OP.

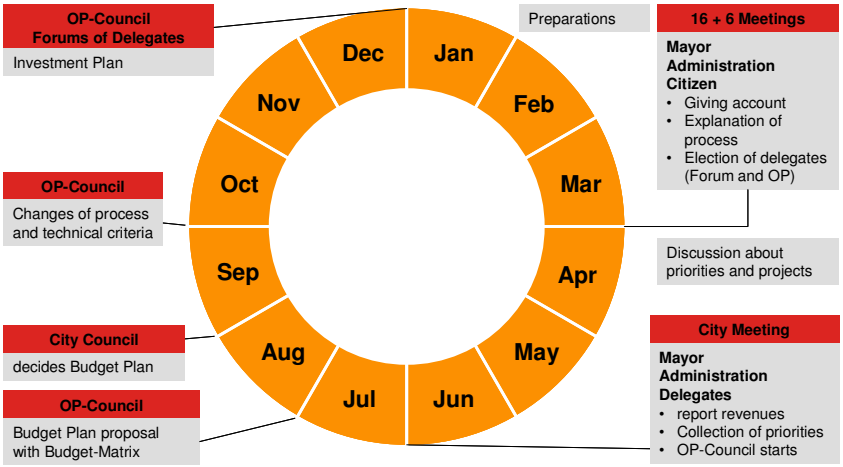


Figure 5. Time schedule of the OP.

The time schedule of all these events is displayed in figure 5.

There are two important aspects in the OP, which we have not mentioned in detail. First, the OP-council has to reflect the process and the technical criteria in October. Adjustments can be made to improve the process. This could be seen as an implemented quality management.

Second, the accountability of the mayor and the administration should be emphasized. The first thing the mayor does in a district meeting is to give account about the last year investment plan.

2.5 The OP as a formal mathematical model

Consider the inhabitants of Porto Alegre each belonging to one of 16 districts and each having an opinion about the public demands. Each inhabitant may revise his opinion when he hears the opinions of others during neighbourhood or district meetings or just as gossip in the streets. At one point in each district, the individual opinions have to be aggregated to the opinion of the district. And at another point the opinions of the district have to be aggregated to the city wide opinion about the public demand.

Thus, **opinion dynamics** happens in the process of OP

- on the district, thematic and the city meetings in a more formal style where every one gets a feeling for the opinions of all others, and
- during the first preparations and between the district and the city meeting (in an informal respectively gossip style).

And after that **social choice** takes place

- in the voting system district delegates may use to aggregate different priorities of micro-regions, and
- in the voting system that computes a city wide priority list out of the district lists.

3. The Social Choice Problem

At first we want to treat the social choice aspects of the OP because they are part of the rules which have to be established in front of any opinion dynamics.
The classical social choice problem is quite the same as in the OP: Given ballots of m voters each containing a total ranking of n candidates, how one should compute an aggregated ranking of all ballots that reflects the common will of the voters?

There is a huge amount of mathematical, economical and philosophical literature about the paradoxes of this problem. So, we will only cover aspects most related to our issue.

Although in the classical social choice problem and in the OP voters are asked for a **ranking** of alternatives the budgeting issue should lead to an allocation of money to the alternatives.

Box 4. The Gibbard-Satterthwaite Theorem

Each voting system which ranks more than two alternatives is either

- dictatorial (choosing automatically the ranking of one specific voter),
- determines categories that can not win, or
- is manipulable (that means that there are conditions under which a voter who knows all other ballots can ensure the winning of his favorite category by voting not his true preferences; this is called tactical voting).

This is a generalization of Arrow's famous impossibility theorem.

Thus, the voters could be asked directly for an **allocation** of money to the alternatives. Obviously, one would only need the percentages of money not the absolute values because the total amount of money to distribute is a constant in each year. In the following we will distinguish ranking ballots and allocation ballots and argue that to some extend allocation ballots would be more appropriate to the budgeting issue.

The ranking ballot forces the voter to decide on a total ranking, he can not decide two alternatives to be equal or an alternative to be three times more important than another. While this restriction of choice is obviously not necessary in a budgeting question there will be some deeper arguments for it.

The use of allocation ballots has familiarities to the voting system used for communal elections in some German federal states, where voters can split their votes and are allowed to cumulate them ('Kummulieren und Panaschieren').
There are two leading questions for the social choice problem:

- What is the equitable aggregated ranking?
- What is the right aggregated ranking for the city?

Box 5. The Borda Count

The Borda-Count maps the ranks into points. Given the setting of the OP one list of priorities would give 13 points to the top category, 12 points to the second … and 0 points to the last. It goes back to the French mathematician, scientist and noblemen Borda in 1770 when it was in contrast to the method of the Marquis de Condorcet. He answered to the potential of tactical voting: "My scheme is only for honest men". A variation with other points is used at the Eurovision Song Contest.

While the first regards the ballot as an individual preference, the second regards the ballot as an opinion about the public demand. While classical social choice theory focuses the first, we argue that the second should be stressed, too. The Gibbard-Satterthwaite theorem (see Box 4) is the key result of mathematical social choice theory and states that all acceptable voting systems are vulnerable for **tactical voting**.

We call the voting system of the OP a (truncated) Borda-Count (see Box 5). Tactical voting strategies for the Borda Count are **compromising** and **burying**. An example for both strategies is a voter who gives Education an upper but not top

rank and Habitation a lower but not the lowest rank. This voter may fear a trend that favours Habitation and forgets Education. This may lead him to the strategy to compromise and give Education the first place instead of his own top rank and to bury Habitation at the bottom of his ballot.

These strategies can be played even more drastic with allocation ballots, which might be one reason to restrict the choice of allocations as it is done by the Borda Count. Another influence on the outcome from the voting system comes in by the truncation. Box 6 shows an example of the real world.

Tactical voting dodges the idea of social choice as mechanism to aggregate the equitable ranking, but there is no way to avoid it. Thus, the social choice problem in the strict sense of aggregating the invulnerable will of the individuals is unsolvable. This may discourage one to use participatory budgeting.

But if we stress our second view point we may get encouraged. We can see the ballot as an opinion about the public demand. The encouraging idea behind this is the so-called **'wisdom of crowds'** (Surowiecki 2004). Several experiments show that the average of group estimates of an unknown quantity (e.g. marbles in a glass) is much better than nearly every single estimate. Under this view point participatory budgeting becomes a great **opinion pool** which does a common judgement about the public demand.

Under this consideration the allocation ballots seem much more appropriate, because it is much more natural to average allocations (by a simple arithmetic mean) than rankings (where a hands-on method is lacking). Tactical voting will correspond under this view point to intended biasing of the own opinion.

Box 6. Aggregated rankings OP 2005

Data from table 1.

	OP (truncated Bo. C.)	**Borda Count**
1.	Habitation	Habitation
2.	Education	Education
3.	Health	Health
4.	Social Assistance	Social Assistance
5.	Water & Seweage	Asphalting
6.	Asphalting	Econ. Dev. …
7.	Econ. Dev. …	Water & Sewage
	…	…

Relying on the wisdom of crowds depends on the independence of the citizens in making their opinion, which is obviously not the case in participatory budgeting due to opinion dynamics in meetings, gossip or through mass media. This comes due to the fact that specifying of a budget is not only a problem of finding the unknown but correct answer, but also a task of negotiating of citizens with other interests. But nevertheless one should give the idea of the wisdom of crowds a bit of attention. The main profit of this view is to raise the scope for the participants form the individual level to the public level.

The possible opinion dynamics lead us to the issue of social consensus. If the opinion dynamics generated by the process of participatory budgeting create an impact on fostering a social consensus (which means that all people have the same opinion about the public demand) the unsolvable social choice problem can be omitted.

Having all this in mind we want to do a simulation analysis how opinion dynamics may foster social consensus in budgeting issues. Under the scope of social consensus it is more appropriate to use allocation ballots.

4. Opinion Dynamics

The following models of opinion dynamics are very rough and neglect several aspects of real discussions. We try to claim and fix some characteristic properties of opinion dynamics and run simulations with artificial citizens. We will look for qualitative not for quantitative statements.

An opinion in budgeting is naturally more dimensional with the number of categories as dimension, thus an opinion is a vector of nonnegative real numbers. Due to the announcement of a fixed amount of money to distribute, each vector has to sum up to this amount. After normalizing the sum of components to one, we have all **nonnegative sum-one-vectors** as our **opinion space**.

The interesting property of continuous opinions is that compromising is possible by averaging (in contrast to a yes-or-no opinion). Lehrer and Wagner (1981) gave an axiomatic argument that the arithmetic mean is the only appropriate to aggregate opinions about an allocation.

Now let's imagine *m* citizens, each having an opinion in the *n*-dimensional nonnegative sum-one-opinion space. Each citizen is willing to revise his own opinion if he discusses with other citizens.

The next assumption about our citizens is that they have **bounded confidence**, which means a citizen only takes opinions into account that are not farther then the bound of confidence ε from their own opinion. To measure a distance in the multidimensional space we take the standard Euclidean distance.

Jager and Amblard (2005) derive the bound of confidence from **social judgement theory** which is a subject of social psychology and says roughly that one judges the opinion of another person by the difference to the own opinion. If it is close one tends to assimilation, otherwise to rejection. We see it also as the willingness to negotiate with not to different citizen.

What is lacking now in our model is the **communication regime**. We want to distinguish gossip and repeated meetings.

Simulation of **gossip** opinion dynamics goes back to Weisbuch et al (2002) and Deffuant et al (2002) (not under this name). In each time step two randomly chosen citizen meet. They compromise in the middle if their distance in opinion is below ε.

Simulation of **meeting** opinion dynamics goes back to Hegselmann-Krause (2002) (not under this name). In each time step each citizen takes all opinions in his area of confidence (the ε-region around his opinion) and builds the arithmetic mean as his new opinion. Thus, he has to know all opinions – it's a meeting.

Obviously both communication regimes are not the real ones, but we believe that real communication structure lies somewhere in between.

First, we want to regard the simplest case to see how these dynamics work. So we consider only two categories. Thus, opinion dynamics takes place on the line from (0,1) to (1,0).

Figure 6 shows on the left hand side an example process for gossip and meeting dynamics with 200 citizens with random and equally distributed opinions in the opinion space. The time walks on the t-axis. Each colour represents the opinion of one citizen evolving in time. We draw three confidence areas in each subfigure to give an impression how far the confidence of a citizen reaches.

A quick description how dynamics work: Generally citizen will move to opinion regions with a high density of citizens. Thus, in our case of uniform distribution the dynamic starts at the border of the opinion space with citizens moving towards the centre, creating a higher density there and thus attracting other citizens even from the centre.

Thus, we see the evolution of opinion clusters. In the following we are interested in how the number, size and location of clusters depend on the bound of confidence and on the number of categories. A social consensus would be if we get only one big cluster.

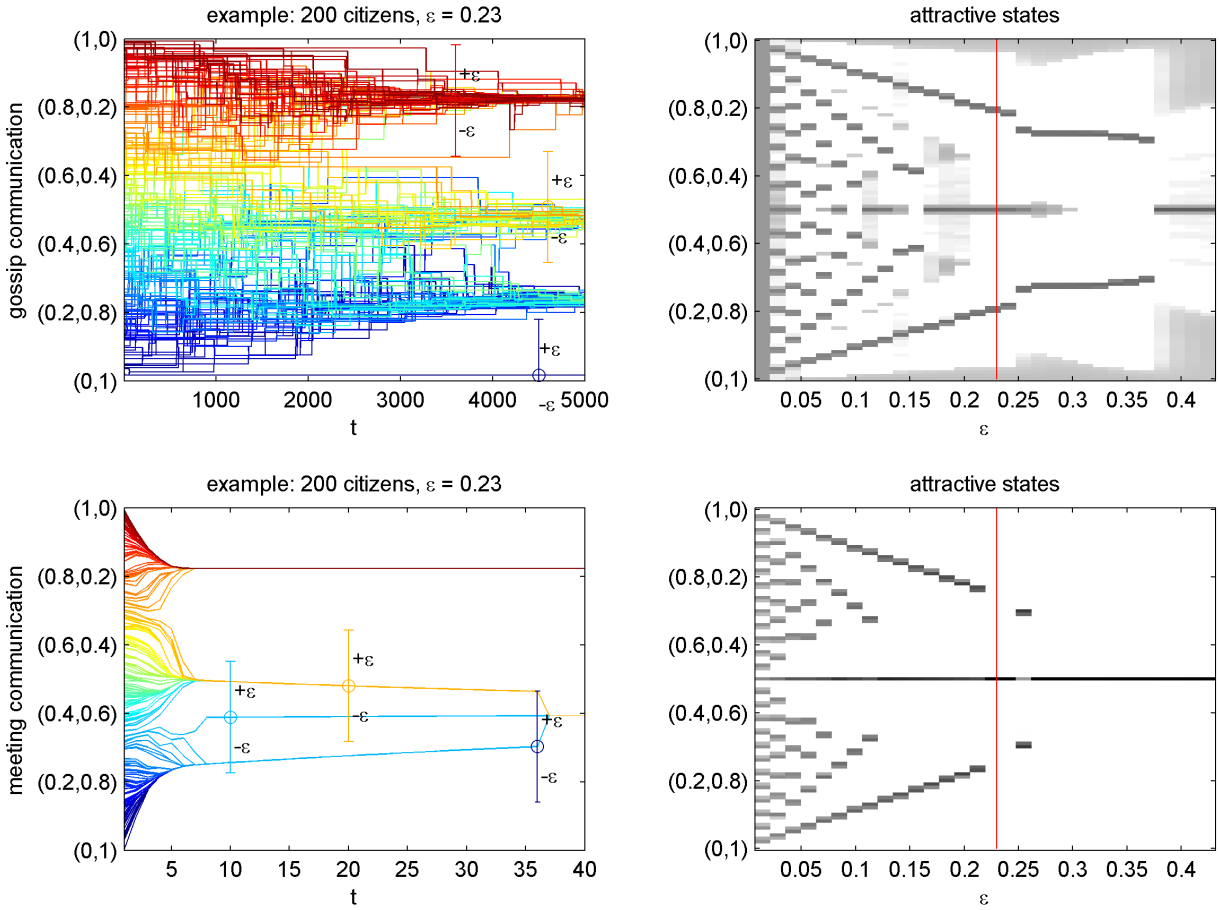


Figure 6. Overview about dynamics with only two categories.

Now, we study the bound of confidence by diagrams of attractive locations for opinion clusters at the right hand side of figure 6. The graphics are derived from interactive Markov chains of the models (see Lorenz (2005)). They give a good overview about the attractive locations of clusters for uniform distributed initial opinions. The x-axis is now ε. The red lines stand for the ε-value of the example. The gray scale stands for the size of the clusters. Black is big.

We will give a **summary about the two category case**. The results can be derived with careful inspection of figure 6.

- In both models there are ε-intervals where a specific number, size and location of clusters are characteristic.
- Gossip dynamics leaves minor clusters at the extremes and for several ε-values also between the major clusters.
- If we neglect the minor clusters under gossip dynamics and compare the number of major clusters to the meeting dynamics, we see that we get to consensus for significantly lower bounds of confidence.
- We observe a very strange 'consensus strikes back'-interval in meeting dynamics (around ε=0.23). But it is not visible in the figure that consensus is very unstable (thus, we need a very uniform distribution of citizens), and it happens only due to very long convergence time (for details see Lorenz (2006)). The example process in figure 6 shows that the long convergence time happens due to one citizen sitting between two clusters. To reach consensus in the example one citizen should have sat between the upper and the central cluster, too.
- Long convergence time is also reported when ε is close to the bifurcation from consensus into polarization.

For the exploration of higher numbers of categories the visualisation gets difficult. Thus we have to focus on 'meta-data' like the average number of clusters, the average size of the biggest cluster and the distribution of cluster sizes. We have explored the parameter space from n=2,…,8 with 200 simulation runs for each ε-value and always 200 citizen with random and uniformly distributed initial opinions.

Figure 7 shows the meta-data and the average convergence time.

Figure 8 shows the distribution of cluster sizes for n=2,3,4 and 8. Black is a big amount of clusters of that size.

We summarize the impact of the number of categories. The results can be derived by careful inspection of figures 7 and 8.

- Raising the numbers of categories leads to more minor clusters not only under gossip dynamics. They also appear in meeting dynamics.
- If we regard the existence of a cluster with a vast majority of citizens (e.g. more than 75%) as a majority-consensus we can say that majority-consensus can be reached for lower bounds of confidence with rising number of categories. But the sinking of the threshold slows down. The impact from 2 to 3 is higher as from 7 to 8.
- The ε-interval between majority consensus and total plurality of opinions is getting shorter with rising number of categories. Thus there may be a surprisingly high impact in getting a little more confident for high numbers of n.
- The convergence times for meeting communication give a hint that reaching majority-consensus may last long if we are only a little bit above the ε-threshold to polarization under even higher number of categories.

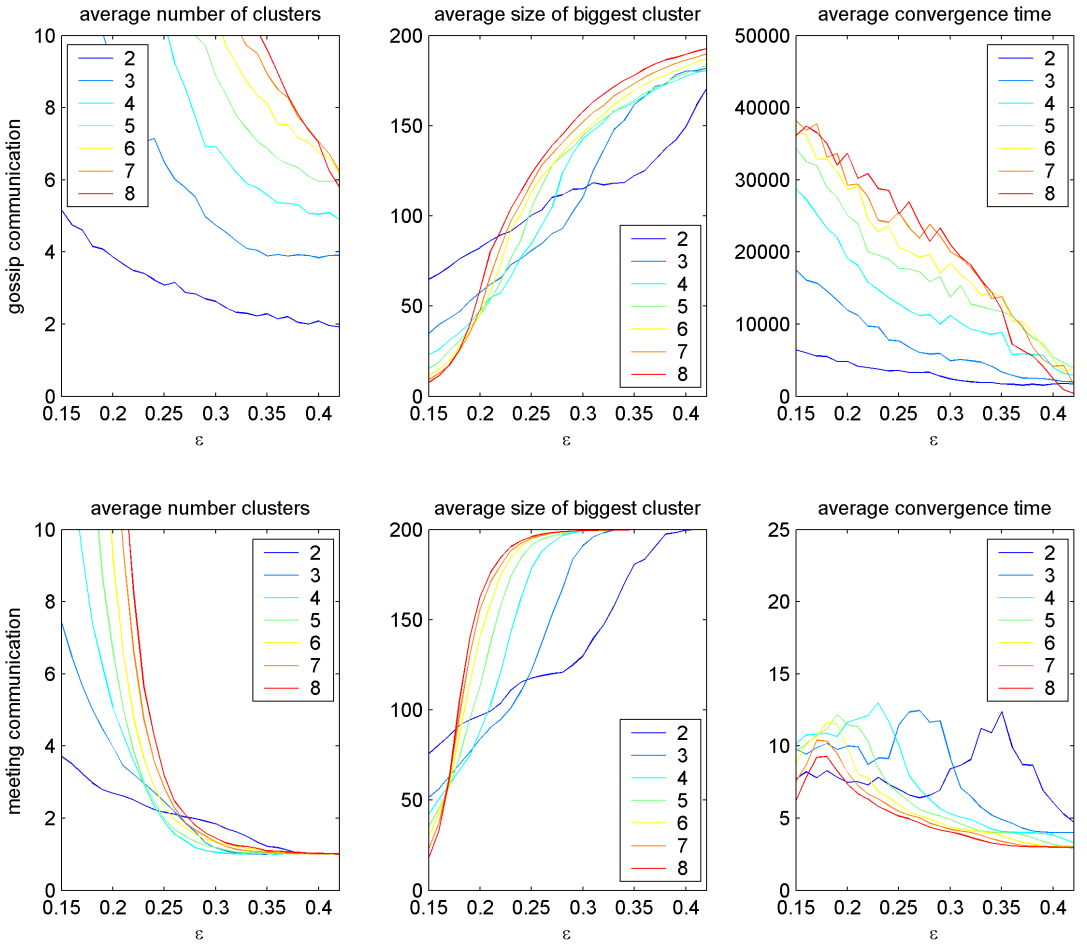


Figure 7. Average number of clusters, sizes of the biggest cluster and convergence times.

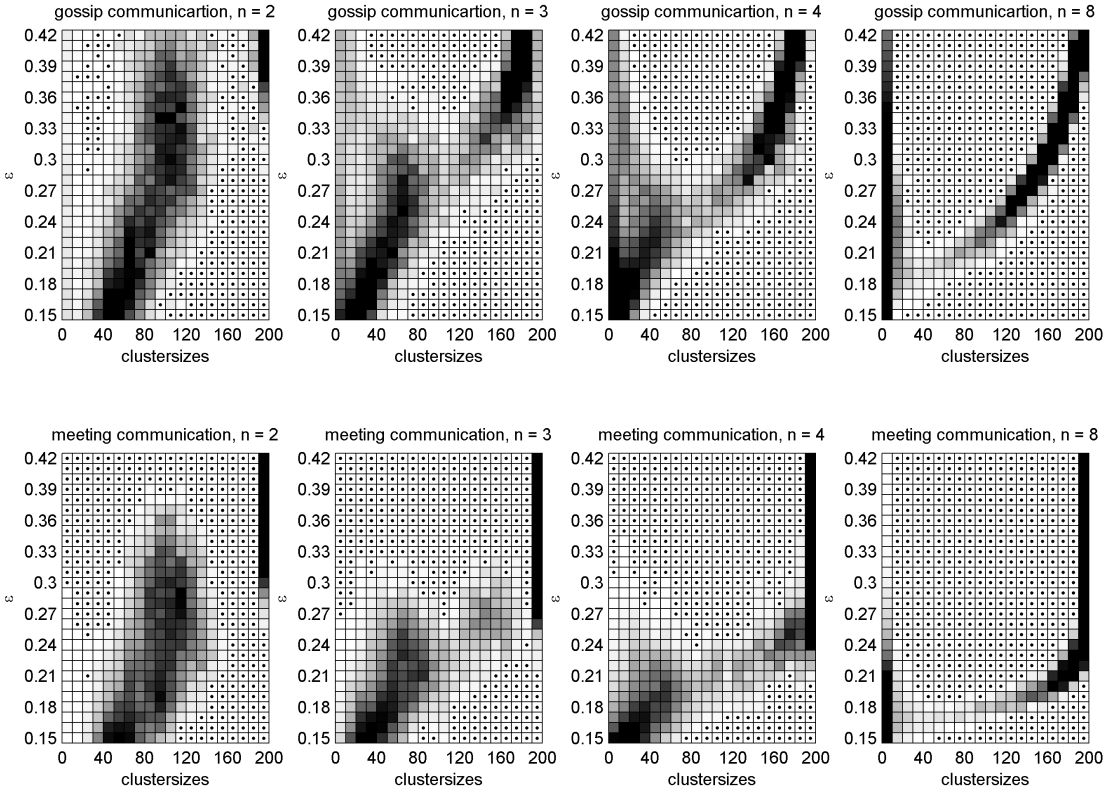


Figure 8. Distribution of sizes of clusters.

Overall we **summarize some qualitative results** which may be useful in the design of participatory budgeting under the aspect of fostering social consensus:

- If we have a number of unorganized citizens with different opinions and bring them into opinion dynamics there will be an automatic formation of certain interest groups only by bounded confidence (and not necessary by big propaganda).
- If we drive opinion dynamics by several meetings where everyone hears the opinion of everyone we have much better chances for reaching total consensus (even for a lower bound of confidence) than if we only trust in gossip opinion dynamics.
- But even if we manage several meetings we run into the danger of very long convergence to consensus.
- Gossip dynamics tend to disconnect some small from big clusters.
- Higher numbers of categories and sum-one-opinions produce for lower bounds of confidence a more diverse clustering. There is no polarization but more plurality.
- If it is possible to raise the bound of confidence for all citizens, it would have more impact to do it at the very beginning to avoid early clustering.
- A vast majority may easier form a big cluster if we give them more categories to decide on. But the effect is getting lower and lower with rising number of categories.
- The drawback of raising the number of categories is that we open more space for minorities to survive in the extremes of the opinion space.

Finally, we want to mention that this model is totally a bottom up approach with very few assumptions on citizens, which does not contain any political forces or entities. To cope with reality and try to get some benefit of these basic simulation results in relation to other questions of participatory budgeting is due to the next section.

5. Further Challenges for Mathematical Results

Considering specifically the studies on Participatory Budgeting in Porto Alegre, most of them tended to explain participation as an instrument for strengthening democracy in local spheres, rather than a method for reaching consensus among citizens with distinct opinions. Sociologists and Political Scientists who have studied OP were mostly interested in analyzing to what extent participation could effectively transform politics in order to better combat corruption, educate citizens and bring policies closer to the people's demands. In other words, the paradigms faced by Social Sciences usually gave low credit to the fact that there can be several methods for citizens to organize their demands (the social choice problem) and that individuals' opinions can change depending on the participation strategy they use (the opinion dynamic paradigm).

Nevertheless, this paper has focused so far only on these two aspects of participatory budgeting processes, and it might be useful to enumerate some other paradigms and political barriers that might be linked to the mathematical results.

5.1 The Problem of Representation and Capture

Representation in participatory budgeting should never be underestimated. Various reasons have been used to condemn participatory processes of not been representative. For instance, citizens may have problems with the dates or certain well organized group may mobilize too many participants and dominate a meeting. Thus, decisions made on the meeting may represent a very biased will.

Problems related to representation are very well known, and typical contra-arguments can be found through many examples as well in the classical system of representative democracy. Some of those are: restrictions due to 5%-clause in Germany, the majority rule in parliaments (which represents only half of the population but rules all power), and sinking voter participation.

Processes of participatory budgeting are also vulnerable to the capture of certain interest group – the same way as it happens in the parliamentary system. Over-representation of self privileging elites or heavy attendance by certain social groups may imply several negative consequences for the participatory process, including difficulties in achieving consensus and even corruption.

Analyzing the case of Salvador, where OP was implemented in 1993, Boschi (1999) describes how the whole participatory process has been contaminated by bureaucrats or partisan groups which were somehow connected to the local Government. Salvador, according to the author, didn't have a strongly organized civil society by the time OP was created. Moreover, the city had a serious paternalist tradition and the approval of the mayor (Lídice Matta) was hitherto very low among city councillors. As a consequence, meetings and decisions were negatively influenced by politically oriented groups, and population got frustrated with the final results of participatory budgeting.

The problem of capture can also benefit strong social and local movements that are against the government. Politically articulated groups that are historically organized are more likely to occupy the newly opened forums as new spaces for demonstrating their insatisfaction.

What happens in the end is that these groups end up forming a barrier to negotiation and opinion dynamics. Because they sustain radical positions and because their political interests go much beyond the scope of a regional or microregional meeting, they make consensus at regional level much more difficult. From our mathematical results we can heavily emphasize that a group coming as a cluster already into a participatory budgeting process do have a bad impact on both the chances of social consensus and the working of the wisdom of crowds.

Therefore, it is worth to think about the implementation of some mechanisms of control. In the sense that citizens should have to bring some evidence that they cover the whole population. Some especially in Germany think of a random invitation of citizens.

5.2 The Problem of Power Balance

Participatory Budgeting is a very powerful bargaining instrument for the Executive, which manages the whole deliberative-participatory process with almost no interference of the Legislative. Because OP is a city government's program, it is technically controlled by secretaries linked to the mayor and his coalition. Besides, as it is well known, OP is a program based on collecting and attending people's demands through direct participation.

However, as it is also known, responding people's demands is also the role of the city councillors, who are the "official representatives" of the people. Prevented to participate in the OP process and constrained before the population's decisions, city councillors claim to be "double losers" with participatory budgeting. As a result, the political balance between Executive and Legislative is affected, benefiting the mayor's side.

Márcia Dias (2000) analyzed the embarrassment phenomenon in Porto Alegre in the period between 1992 and 1999. The author argues that councillors in the opposition had increasing difficulties in leading with the population after OP was implemented. Embarrassed to defend policies that might go against the ones voted by the population, councillors started loosing power inside the City Chamber. They felt bounded to vote with the government even in issues that were not directly related to OP.

We now know that city councillors must somehow participate in the meetings and that their demands have to be included in the final set of priorities. There must be some negotiation strategy in order to merge the direct interests of the people and the interest of their official representatives. One new role to be filled by the city councillor might be to control and ensure a balanced participation of demographic groups. But that will be a totally new job, were objective tools and a political culture have to be invented for.

5.3 The Problem of Institutionalisation

What is the actual interest of public authorities in improving wide open meetings that enable everybody to hear everybody? As we have seen above with mathematical estimates, in meetings where everyone hears the opinion of everyone there are better chances for reaching general consensus. However the reason

why government doesn't promote several of these meetings is not only financial. The OP has never been established by law. Similarly, there was no legal claim for citizens to get involved. Even the inventors of the OP claim that institutionalization of OP is certainly one of the most controversial aspects of this policy.

In Porto Alegre, for instance, the institutionalisation has been one of the major claims of the Workers' Party (PT) during its first mandate (1989-1992). The party feared that, in case the opposition took the power in 1993, the program would be abolished. However, in 1994 – now during Tarso Genro's Government –, OP had already become one of the main flags of PT not only in Porto Alegre but also in every other city the party had administrated. As an instrument of partisan propaganda, OP was being continuously used by PT to convince the electorate that opposite parties would threat the "participatory atmosphere" that the city had achieved if they were elected.

What happened next was a turn-table: the parties in the opposition decided to support the institutionalisation, arguing that it would prevent PT of making a political use of the participatory budgeting program (DIAS, 2000).

One aspect we can contribute from our social choice considerations is that whether one wants to institutionalise participatory budgeting one should not do it in the hard way, like getting the regimento interno (see references) by law, but by ensuring the self-adaptation of the process to its aim every year again. By adapting the rules one might circumvent the participants to concentrate on learning tactical voting then on thinking and negotiating about the public demand.

6. Conclusion

To conclude we mention the key results and put some aspects into a broader picture.

- **Allocation vectors** fit much better to the issue of budgeting than rankings. But nevertheless rankings restrict the opinion which might restrict the effects of possible tactical voting. One should very carefully decide on how citizens should express their opinion and how they were asked about their opinion and demands. E.g. they should also be asked to think about the public demand not only about their individual demand, because the total equitable aggregation of all individual desires is structurally impossible.
- If we see participatory budgeting as a method to bring some opinion dynamics into civil society to **foster social consensus** one is right to **raise the number of categories** to decide on, but perhaps it not necessary to go beyond eight, because the fostering effect slows dramatically down.
- It is well worth to think carefully about social choice considerations and opinion dynamics, especially if one dreams of getting advanced participation not only to grassroots issues but also for a city, state and federal issues; as Santos (1998) states **"From Technobureaucracy to Technodemocracy"**.
- The institutionalisation of participatory budgeting should not destroy the **self-regulating mechanism** and be careful to create it not as another city council, but more as a new opinion pool with ever developing decision rules. Tools to measure and control this must be invented.
- As sometimes stated, the civil society is getting or should get the **forth force** in democracy (additional to Legislative, Executive and Judicative). Participatory budgeting is a first experience to establish it as a democratic tool which can perhaps use the wisdom of crowds and may be used for several positive side effects as lowering corruption, educate people in citizenship and fostering social consensus. The implementation of OP thus could make a counterpart to the growing number of not really democratic NGOs which claim to represent social society.

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